Two approaches to the semantics of questions: Sets of Answers vs. Partition Semantics

1 What kind of semantic objects are questions?

- A basic tenet of Formal Semantics is that meaning can be equated with truth-conditions. Yet direct questions such as (1) and (2) cannot be said to be true or false. They do not express propositions. Rather, they express requests for propositional information.

(1) Which guests showed up at the party?
(2) Did Mary go to the party?

Before deciding what the denotation of questions is, let us look at what kind of empirical evidence can be used in order to theorize about the meaning of questions. There are at least two kinds of evidence that are readily available:

1. One understands a question when one knows what counts as an answer to it.

2. Questions are not necessarily direct questions. They can also be embedded, as in (3), in which case they contribute to the sentence’s truth-conditions.

(3) a. Jack wonders whether Mary went the party  
b. Jack asks which guests showed up at the party  
c. Jack knows whether Mary went to the party  
d. Mary told Jack which guests showed up at the party  
e. Peter and Mary agree on who they should invite

1.1 Questions and Answers

- Polar Questions

(4) a. Did Mary show up at the party?  
b. (i) Yes (, she did) / No (, she didn’t)  
   (ii) Everybody who was invited came to the party  
   (iii) Mary came to the party, but Jack didn’t  
   (iv) ?(No,) Jack did.  
c. (i) #Yes, Jack did  
   (ii) #Mary is Jack’s sister
(iii)#It is sunny outside
(iv)#Mary came to the party, and it is sunny outside

- **Constituent Questions (aka wh-questions)**

(5) Which guests showed up at the party?
   a. #Yes / # No
   b. #It is sunny outside
   c. Peter and Mary did
   d. At least Peter did
   e. Peter and Mary came, and nobody else did
   f. Peter or Mary came (I don’t know which)

- **Alternative Questions**

(6) Did Sue marry Jack or Bill?

   a. Polar reading (calls for a yes-no answer): Is it the case that Sue married either Jack or Bill?
   b. Alternative Reading: Which of Jack and Bill did Sue marry?

Insertion of ‘either’ blocks the alternative reading (cf. Larson 1985):

(7) Did Sue marry either Jack or Bill?

1.2 **Embedded Questions**

(8) Jack knows whether Mary showed up at the party

   ~If Mary showed up at the party, Jack knows that Mary showed up at the party, and if Mary did not show up at the party, Jack knows that she didn’t.

(9) Jack and Peter agree on whether Mary showed up at the party

   ~Either both Jack and Peter believe that Mary showed up at the party, or both believe that she did not.

(10) Jack knows which guests went to the party

   ~For every guest x who went to the party, Jack knows that x came to the party (, and for every guest x who did not go to the party, Jack knows that x did not go to the party).

(11) Jack wonders which guests went to the party

   ~~Jack does not know which guests went to the party and wants to know which guests went to the party.

1.3 **Basic Goal of a Formal Semantics theory of interrogatives**

Associate to every question a semantic object on the basis of which one can offer a general characterization

a) of what counts as an answer to the question, and b) of the truth-conditions of declarative sentences in which an interrogative clause is embedded.

In this first class, we will present in informal terms two prominent proposals regarding the semantic denotations of polar questions, constituent questions and alternative questions, and discuss how they relate to these two goals; we are not going to deal with the way these denotations are derived compositionally (but see next classes).

The two types of proposals we are going to discuss are a) proposals in which the denotation of a question is viewed as a set of answers (Hamblin 1973; Karttunen 1977) and b) so-called Partition Semantics, as developed by Groenendijk and Stokhof in various publications (Groenendijk and Stokhof 1982, 1984a,b, 1990).
One of our goals in this course is to compare the relative merits of these two approaches. Today, we’ll see some reasons to favor Partition Semantics, but in the next classes additional arguments will lead us to reconsider this conclusion.

2 Questions as denoting sets of true answers – Karttunen (1977)

2.1 Hamblin Denotations – cf. Hamblin (1973)

- A convention (when talking informally): If S is a declarative sentence, let us use ‘That S’ to refer to the proposition expressed by S (a semantic object – a set of possible worlds –, not a linguistic expression).
- Hamblin’s proposal is to associate to every question the set of the propositions that are expressed by its possible answers, in a restricted sense of ‘possible answers’.

Hamblin denotation for polar questions:

(12) Did Mary come?
    \sim \{That Mary came, That Mary did not come\}

(13) ?p \sim \{that p, that not-p\}

Hamblin denotation for constituent questions:

(14) Who came?
    \sim \{That Jack came, That Mary came, \ldots\} = \{That x came: x is a human and x \in D\}

(15) Which books did Jack read? \sim \{That Jack read x: x is a book\}

Hamblin denotation for alternative questions:

(16) Did Sue marry Jack or Paul? \sim \{That Sue married Jack, That Sue married Paul\}

(17) ?(p or q) \sim \{p, q\}

2.2 Karttunen Denotations – cf. Karttunen (1977)

- Hamblin denotations are (more or less) constant across worlds.
- Karttunen: In a given world \(w\), a question denotes the set of its answers that are true in \(w\).
- For any question \(Q\), let us note \(\text{Hamblin}(Q)\) its Hamblin denotation.

(18) Karttunen’s proposal: \(\|Q\|(w) = \{\phi : \phi(w) = 1 \land \phi \in \text{Hamblin}(Q)\}\)

In other terms:

(19) \(\|Q\| = \lambda w_1.\lambda \phi_{<s,t>}.(\phi(w) = 1 \land \phi \in \text{Hamblin}(Q))\)

Semantic Type of Questions: \(<s,<<s,t>,t>>\)

Karttunen’s Semantics for Polar Questions:

(20) Did Mary go to the party?
    \sim \text{The function } f \text{ such that:}
    - \(f(w) = \{\text{That Mary went to the party}\}\) if ‘Mary went to the party’ is true in \(w\)
\[-f(w) = \{\text{That Mary did not go to the party}\} \text{ if ‘Mary did not go to the party’ is true in } w\]

Karttunen’s Semantics for Alternative Questions:

(21) Did Sue marry John or Jack?
\[\neg f(w) = \emptyset \text{ if, in } w, \text{ Sue did not marry either John or Jack}\]
\[-f(w) = \{\text{That Sue married John}\} \text{ if, in } w, \text{ Sue married John and did not marry Jack}\]
\[-f(w) = \{\text{That Sue married Jack}\} \text{ if, in } w, \text{ Sue married Jack and did not marry John}\]
\[-f(w) = \{\text{That Sue married John, That Sue married Jack}\} \text{ if, in } w, \text{ Sue both married John and married Jack}\]

Karttunen’s Semantics for Constituent Questions:

(22) Which boys did Mary invite?
\[\neg f(w) = \{\phi : \phi(w) = 1 \land \exists d (d \in \{\text{boys}\}(w) \land \phi = \text{That Mary invited } d)\}\]

2.3 Complete Answers and Embedded Questions in Karttunen’s system

2.3.1 Karttunen’s Notion of Complete Answer

The complete answer to a question Q in a world w is the conjunction of all the members of Q(w), i.e. the conjunction of all the elementary true answers. Let us thus define:

(23) a. \(\text{Ans}_K(Q)(w) = \lambda w. \forall \phi (\phi \in Q(w) \rightarrow \phi(w) = 1)\)

b. For short: \(\text{Ans}_K(Q)(w) = \bigwedge Q(w)\)

Constituent Questions

(24) Which boys came?

Suppose that there are four boys, Jack, Peter, Alfred, and Ken, and that Jack and Peter came but Alfred and Ken didn’t. Then the complete answer to (24) is the proposition that Jack and Peter came. So the complete answer, in Karttunen’s sense, does not contain any negative information. The complete answer to (24) is thus completely distinct from the complete answer to (25):

(25) Which boys did not come

In the same situation, the complete answer to (25) is the proposition that neither Alfred nor Ken came.

At first sight, this seems to account for our intuitions about what counts as a fully cooperative answer to a wh-question:

(26) a. Which boys came?

b. Jack and Peter (did).

(27) a. Which boys did not come?

b. Alfred and Ken (didn’t come).
However, note that both the answers in (26-b) and (27-b) tend to be interpreted as conveying the following:

(28) Jack and Peter came, and Alfred and Ken did not come.

We say that such answers are interpreted as exhaustive. We would like to understand why (see below).

- What if no boy came?

The complete answer to (26-a) in a world w in which no boy came is simply the tautology:

- $K(Q)(w) = \emptyset$ in this case.
- $\text{Ans}_K(Q)(w) = \lambda w. \forall \phi (\phi \in Q(w) \rightarrow \phi(w) = 1)$
- $= \lambda w. \forall \phi (\phi \in \emptyset \rightarrow \phi(w) = 1)$
- $= \lambda w. (\text{the tautology})$

Alternative Questions

(29) Did Sue have dinner with Jack or Peter

The complete answer to (29) can be:

- a) the tautology in a world in which Sue didn’t have dinner with either Jack or Peter
- b) the proposition that Sue had dinner with Jack (resp. Peter) in a world in which she had dinner with Jack but not with Peter (resp. with Peter but not with Jack)
- c) the proposition that she had dinner with both Jack and Peter if she in fact had dinner with both.

In fact, as has been observed by various authors, a question such as (29) strongly suggests that Sue had dinner with one and only one of the two, a fact that calls for an explanation. As a result, only the b)-clause is relevant.

2.3.2 Embedded Questions under $\text{Know}$

- Goal: characterizing the meaning of $\text{know + question}$ in terms of the meaning of $\text{know + that-clause}$

Basic Intuition: $\text{Jack know Q}$ is true in w just in case Jack knows that p in w, where p expresses the complete answer to Q in w.

(30) $\text{know}_{\text{that}}$

a. Semantic type: $<< s,t>,< s,< e,t >>>$ – $\text{know}_{\text{that}}$ takes a proposition (type $< s,t >$) and returns a property (a function from worlds to sets, type $< s,< e,t >>>$)

b. $\llbracket \text{know}_{\text{that}} \rrbracket = \lambda \phi_{s,t}, \lambda w_s, \lambda x_e : \phi(w) = 1$. every world w’ compatible with x’s beliefs in w is such that $\phi(w’) = 1$

(31) $\text{know}_{\text{?}}$ (preliminary version)

a. Semantic type: $<< s,<< s,t>,t >>>,< s,< e,t >>>$ – $\text{know}_{\text{?}}$ takes the denotation of a question, i.e. a function from worlds to sets of propositions (type $< s,<< s,t>,t >>>$) and returns a property.

b. $\llbracket \text{know}_{\text{?}} \rrbracket = \lambda Q_{s<t}, \lambda w_s, \lambda x_e. \llbracket \text{know}_{\text{that}} \rrbracket (\text{Ans}_K(Q)(w))(w)(x) = 1$

However, this predicts that a sentence such as $\text{Jack knows who came}$ is automatically true if nobody came. This is so because if nobody came, then the denotation of $\text{who came?}$ in the actual world is the tautology, so that $\text{Jack knows who came}$ is true if Jack knows that the tautology is true.

So a modification is needed. In a nutshell:

(32) ‘Jack knows Q’ is true in w just in case Jack knows that $\text{Ans}_K(Q)(w)$ is true and, if $\text{Ans}_K(Q)(w)$ is the tautology, he knows that $\text{Ans}_K(Q)(w)$ is the tautology, i.e. that $Q(w) = \emptyset$. 

5
\[
\|\text{know}\| = \lambda Q<\ll<,+,>,+,\rr\lambda w,\lambda x.\\
\|\text{know}_{\text{that}}\| (\text{Ans}_K(Q)(w)(v)) (\ll w\rr) (x) = 1] \land [(Q)(w) = \emptyset \rightarrow \|\text{know}_{\text{that}}\| (\lambda v.Q(v) = \emptyset)(w)(x) = 1]
\]

2.4 Two predictions of Karttunen’s system: weakly exhaustive readings and de re readings

2.4.1 Weak and Strong Exhaustivity

Suppose there are four relevant individuals in the domain, call them Peter, Mary, Sue, and Alfred. Suppose further that both Peter and Mary showed up at the party, and Sue and Alfred did not. Consider then the following sentence:

(34) John knows who showed up at the party.

In such a context, the complete answer to the question *Who showed up at the party?* is the proposition expressed by (35):

(35) Peter and Mary showed up at the party.

Note, in particular, that this complete answer does not entail that only Peter and Mary showed up. For (34) to be true in such a context, according to K.’s proposal, it should simply be the case that John knows that Peter and Mary showed up at the party. His beliefs regarding Sue and Alfred are completely irrelevant in order to assess (34)’s truth-conditions.

More generally, K.’s semantics makes (34) logically equivalent to:

(36) For any individual \(x\) such that \(x\) showed up at the party, John knows that \(x\) showed up at the party.

Intuitively, however, as pointed out by Groenendijk and Stokhof (1982), a sentence such as (34) seems to express something much stronger. In the above context, it seems to entail not only that John knows that Peter and Mary showed up at the party, but also that Sue and Alfred did not.

K. discusses strengthening the truth-conditions of (34) into the following:

(37) a. For any individual \(x\), John knows whether \(x\) showed up at the party
    b. For any individual \(x\), if \(x\) showed up at the party, Jack knows that \(x\) showed up at the party, and if \(x\) did not show up at the party, Jack knows that \(x\) did not show up at the party.

As pointed out by K., this might be done by adding all the true propositions of the form \(x\) did not show up at the party. to the denotation of the question. K. rejects this option, based on the following argument. This revision makes *Who dates Mary?* and *Who does not date Mary?* equivalent, with the result that the two following sentences are predicted to be equivalent, which they are not:

(38) a. Bill wonders who dates Mary
    b. Bill wonders who does not date Mary

Further, K. notes that (39-a) does not entail (39-b), because Bill might know who dates Mary without knowing what the domain of quantification is, in which case he cannot know who does not Bill Mary.

(39) a. Bill knows who dates Mary
    b. Bill knows who does not date Mary

• Weakly Exhaustive Reading

An embedded question is said to receive a *weakly exhaustive reading* if it is interpreted as denoting its complete answer in Karttunen’s sense. The weakly exhaustive reading is the one paraphrased in (36).

\(^1\)K.’s text contains a typo. It says “this is not a desirable result, considering the fact that (40a) and (40b) [K.’s numbering - (38-a) and (38-b) in the present document] intuitively do appear to be synonymous”. Clearly, what was intended was “...intuitively do not appear to be synonymous.”
• Strongly Exhaustive Reading

Groenendijk and Stokhof (1982) argue that the actual reading of constituent questions under know is much stronger than that.

(40) John knows who showed up at the party

Let $S$ be the set of all the people who showed up at the party. Then (40) is true, under the strongly exhaustive reading, just in case John knows that every individual $x$ in $S$ showed up at the party and that no individual outside of $S$ came to the party.

Note that (40) can be true under the strongly exhaustive reading even if John does not know who did not show up: suppose that John is simply ignorant about the domain of quantification. Then he could know, for every $x$ who showed up, that $x$ showed up, know that nobody else came, but having no idea who are the people who did not show up.

2.4.2 De re and de dicto readings

(41) Which linguists showed up at the party

Suppose that the actual world $w_0$ is as follows:

- $\|\text{linguists}\| = \{a, b, c, d\}$
- $\|\text{showed up at the party}\| = \{a, b, e, f\}$.

We have:

(42) a. $\|(41)\|(w_0) = \{\text{That a showed up at the party, That b showed up at the party}\}$
    b. $\text{Ans}_K((41))(w_0) = \text{That a and b came to the party}$

Suppose further that, in $w_0$, Bill happens to know that a, b, e and f showed up at the party and nobody else did. Assume that Bill, however, has no idea who is a linguist and who is not. In such a situation, given K.'s semantics, (43) is predicted to be true even though both sentences in (44) are predicted to be false:

(43) Bill knows which linguists showed up at the party
(44) a. Bill knows who is a linguist who showed up at the party
    b. Bill knows which of the people who showed up at the party are linguists

This is so because the complete answer (in (42-b)) to (41) in $w_0$ does not say anything about who is a linguist and who is not. The complete says, for any $x$ who is a linguist who came, that $x$ came, but not that $x$ is a linguist. Today’s common wisdom is that the reading predicted by K.'s system exists (modulo weak vs. strong exhaustivity), but there is also a second reading which makes (43) equivalent to (44-a) and (44-b).

(45) a. de dicto reading: for any $x$ who is actually a linguist and such that $x$ came, Bill knows that $x$ is a linguist and that $x$ came (and maybe also that no other linguist came).
    b. de re reading: for any $x$ who is actually a linguist and such that $x$ came, Bill knows that $x$ came (and maybe also...)

• A possible amendment meant to capture de dicto - readings in a system à la Karttunen:

(46) a. K.'s actual proposal:
    $\|\text{which linguists showed up?}\|(w) = \{\text{That x showed up: x is a linguist and x showed up}\}$
    Possible modification:
    $\|\text{which linguists showed up?}\|(w) = \{\text{That x is a linguist and x showed up: x is a linguist and x showed up}\}$
3 Partition Semantics


3.1 Partition Semantics

3.1.1 Questions as Equivalence Relations

A question can be viewed as imposing a certain perspective on the ‘logical space’, i.e. the set of possible worlds. More precisely, by expressing a question, a speaker makes clear that she is interested in certain aspects of the world (those that are relevant to the question she asked), and not in others. Let us say that a question Q makes two worlds $w_1$ and $w_2$ equivalent if $w_1$ and $w_2$ are (maybe) different only along dimensions that are not relevant to the question Q. Partition Semantics, in a nutshell, says that the meaning of a question is an equivalence relation over the set of possible worlds.

3.1.2 Polar Questions

(47) a. Did Mary come to the party?
   b. For short: $\mathbf{?}p$

Notation: $p(w)$ the truth-value of $p$ in $w$.

(48) Equivalence relation induced by a polar question:
   For any two worlds $w_1$ and $w_2$, $w_1 \equiv Q w_2$ iff $p(w_1) = p(w_2)$

3.1.3 Constituent Questions

(49) Which linguists came?

Notation: $\text{LINGUIST}(w) = \{x: x \in \|\text{linguist}\|(w)\}$

(50) Equivalence relation induced by (49):
   $w_1 \equiv (49) w_2$ iff $\text{LINGUIST}(w_1) \cap \text{CAME}(w_1) = \text{LINGUIST}(w_2) \cap \text{CAME}(w_2)$

More generally:

(51) a. $[\text{Wh - RESTRICTOR}] [\ldots t_i \ldots]$
   b. $w_1 \equiv (51-a) w_2$ iff $\text{RESTRICTOR}(w_1) \cap \lambda x.\phi(x)(w_1) = \text{RESTRICTOR}(w_2) \cap \lambda x.\phi(x)(w_2)$

3.2 Complete answers

An equivalence relation over logical worlds divides the logical space into a partition of cells. Each cell corresponds to a complete answer to the question. Various notions can be defined in terms of this partition.

Propositions are viewed as sets of worlds. A complete answer is a proposition that fully satisfies the questioner’s request, i.e. locates her in one particular cell of the partition.

(52) $\phi$ is the complete answer to $Q$ in $w_0$ if $\phi = \lambda w. (w \equiv Q w_0)$

Examples

(53) Which linguists came?

(54) Complete answer to (54) in a world $w_0$
   a. $\lambda w. (w \equiv Q w_0)$
   b. $\lambda w. (\text{LINGUIST}(w) \cap \text{CAME}(w)) = (\text{LINGUIST}(w_0) \cap \text{CAME}(w_0))$
Suppose that in \( w_0 \) the set of linguists who came is \{a, b, c\}. Then the complete answer to (54) in \( w_0 \) is the proposition consisting of all the worlds in which the set of linguists who came is \{a, b, c\}. In other words, it is the proposition expressed by:

\[(55) \quad \text{a, b and c are linguists who came and nobody else is a linguist who came.}\]

### 3.3 Extension and Intension of a question

**Intension of a question** According to Partition Semantics, the meaning of question is the equivalence relation it induces, or rather, a function which is the 'curried' version of this equivalence relation: the function from words to sets of worlds which maps every world to the set of worlds which are equivalent to it - i.e. the cell it belongs to, i.e. the proposition that is the complete answer to the question in that world.

\[(56) \quad \|Q\| = \lambda v. \lambda w. w \approx_Q v\]

This is the *intension* of a question: a function from worlds to propositions.

**Extension of a Question** The extension of a question in a world \( w_0 \) is simply the proposition that is the complete answer to the question in \( w_0 \), i.e. the results of applying the intension of the question to \( w_0 \).

\[(57) \quad \text{Extension of } Q \text{ in } w_0 \quad \|Q\|(w_0) = \lambda w. w \approx_Q w_0\]

### 3.4 Embedded Questions in Partition Semantics

#### 3.4.1 Intensional and Extensional Predicates

**Extensional Question Embedding Predicates** They are predicates that combine with proposition-denoting expressions; they can thus combine both with a *that*-clause and an interrogative; when they combine with an interrogative, the interrogative’s denotation is its *extension*, i.e. the proposition that is its complete answer in the actual world.

\[(58) \quad \begin{array}{ll}
\text{a.} & \text{Jack knows that Mary came} \\
\text{b.} & \text{Jack knows whether Sue came} \\
\text{c.} & \text{Jack knows which linguists came} \\
\text{d.} & \text{Jack knows that Mary came and whether Sue came} \\
\end{array}\]

\[(59) \quad \text{Jack knows } Q \quad \sim \text{Jack knows that } \phi, \text{ with } \phi = \text{the complete answer to } Q\]

General Idea: no need for a special clause about *know*. Only one lexical entry is needed:

\[(60) \quad \|\text{know}_{\text{that}}\| = \lambda \phi_{<s,t_\succ}>. \lambda w_s. \lambda x_c : \phi(w) = 1. \text{ every world } w' \text{ compatible with } x's \text{ beliefs in } w \text{ is such that } \phi(w') = 1\]

Embedded Questions can either denote their intension of their extension when they are embedded (this idea can be implemented in various ways). \(\sim\) accounts for coordination facts, among other things.

\[(61) \quad \text{Jack knows who came } \sim \text{true in } w_0 \text{ just in case Jack knows the complete answer to ‘who came?’ in } w_0\]

More formally:

\[(62) \quad \|\text{Jack knows who came}\| (w) = 1 \text{ iff } \|\text{knows}\| (\|\text{who came}\| (w))(w)(\text{JACK}) = 1\]
Other extensional predicates: tell, announce, discover, realize, …

By definition, an extensional predicate always relates an individual to the true answer to the question, even if the predicate is not itself factive (cf. tell)

**Intensional Question Embedding Predicates** Predicates which express a relation between an individual and the intension of a question, rather than a proposition.

(63) wonder, ask,…

Are there Question Embedding Predicates which are neither intensional nor extensional? What about agree on?

(64) a. Jack and Peter agree that Mary won the game
   b. Jack and Peter agree on who won the game.

### 3.4.2 Strongly Exhaustive Readings

G&S predict *strongly exhaustive readings* for embedded questions under extensional predicates.

(65) Jack who came
   a. K. : for any x who came, Jack knows that x came
   b. G&S : Jack knows that X came and that only X did, where X is the set of the linguists who actually came

Note: Partition Semantics does not make (65) equivalent to (66):

(66) Jack knows who did not come

(65) is true in \( w_0 \) if Jack knows the proposition \( \lambda w. (CAME(w) = CAME(w_0)) \). This proposition is not necessarily identical to \( \lambda w. ('DID− NOT− COME(w) = DID− NOT− COME(w_0)) \). There could be two worlds in which exactly the same people came, but not exactly the same people did not come, because the two worlds in question do not contain exactly the same individuals.

### 3.4.3 De re and de dicto readings

- Partition Semantics predicts the *de dicto* reading:

(67) a. Jack knows which linguists came
   b. \( \| Jack \text{ knows which linguists came}\|(w_0) = 1 \text{ iff } \|\text{knows}\| [\lambda w. (\text{LINGUIST}(w) \cap CAME(w) = \text{LINGUIST}(w_0) \cap CAME(w_0))](JACK) = 1 \)

Suppose Jack knows that X and Y came but fails to know that X is a linguist. Then (67) is false.

- The *de re* reading can be predicted by providing a way of interpreting the restrictor *de re*. A straightforward way to do this is to assume that the restrictor contains a separate world argument that can be saturated by a world variable that can either be bound by the attitude verb or by an indexical variable referring to the actual world. We would then have:

(68) \( \| Jack \text{ knows which linguists}_w_0 \text{ came}\|(w_0) = 1 \text{ iff } \|\text{knows}\| [\lambda w. (\text{LINGUIST}(w_0) \cap CAME(w) = \text{LINGUIST}(w_0) \cap CAME(w_0))](JACK) = 1 \)

### References


