1 Proto-Questions and Polar Questions

- Proto-Questions and proto-question operator.

(1) Proto-question operator
a. $\parallel ? \parallel = \lambda \phi_{<s,t>}. \lambda w. \lambda \psi_{<s,t>}. (\psi = \phi \land \psi(w) = 1)$
b. Type of ‘?’: $<s,t>, <s,<<s,t>, t >$

(2) Proto-questions
a. $\parallel ? S \parallel = \lambda w. \lambda \phi_{<s,t>}. (\phi = \lambda v. (\parallel S \parallel (v) = 1 \land \phi = \parallel S \parallel ))$
b. Type of (proto-)questions: $<s,<<s,t>, t >$

- Polar-questions (inspired from Karttunen, but different)

(3) $\parallel O \parallel = \lambda Q_{<s,<s,t>, t >}. \lambda w. \lambda \phi_{<s,t>}. (\phi = \lambda v. (\parallel \parallel ? S \parallel (v) = 1 \land \parallel S \parallel (w) = 1))$

(4) $\parallel O[?S]\parallel = \lambda w. \lambda \phi_{<s,t>}. (\phi = \lambda v. (\parallel ? S \parallel (v) = 1 \land \parallel S \parallel (w) = 1))$

Let us prove that (5-a) and (5-b) are equivalent

(5) a. $\phi = \lambda v. (\parallel ? S \parallel (v) = 1 \land \parallel S \parallel (w) = 1)$
b. $\phi = \parallel S \parallel$ if $\parallel S \parallel (w) = 1$ and $\phi = \parallel - S \parallel$ if $\parallel S \parallel (w) = 0$

- Suppose that $\parallel S \parallel (w) = 1$. Then $\parallel ? S \parallel (w) = \parallel S \parallel$.
Hence $\phi = \lambda v. (\parallel ? S \parallel (v) = 1 \land \parallel S \parallel (w) = 1) = \lambda v. (\parallel \parallel ? S \parallel (v) = 1 \land \parallel S \parallel = 1) = \parallel S \parallel$.
- Suppose that $\parallel S \parallel (w) = 0$. Then $\parallel ? S \parallel (w) = 0$.
Hence $\phi = \lambda v. (\parallel ? S \parallel (v) = 1 \land \parallel S \parallel (w) = 0) = \lambda v. (\parallel \parallel ? S \parallel (v) = 0 \land \parallel S \parallel = 0) = \parallel - S \parallel$

Hence:

(6) $\parallel O[?S]\parallel = \lambda w. \lambda \phi_{<s,t>}. (\phi(w) = 1 \land (\phi = \parallel S \parallel \lor \phi = \parallel - S \parallel))$

2 Alternative Questions

- Basic Disjunction

(7) $\parallel \phi \lor \psi \parallel = \phi \cup \psi$
$= \lambda w. (\phi(w) = 1 \lor \psi(w) = 1)$
If \( \phi \) and \( \psi \) are both of an identical type ending in \( t \), then:
\[
||\phi \circ \psi|| = \lambda x_{1 .. n} . \phi(x_1) \odot \psi(x_1) \odot \ldots \odot \psi(x_n)
\]

(8) We say that an expression is of a semantic type ‘that ends in \( t \)' if its type is of the form
\[
< \tau_1, \tau_2, \ldots, \tau_n, t >
\]

If \( \phi \) or \( \psi \) is of a semantic type ‘that ends in \( t \)' then:
\[
||\phi \circ \psi|| = \lambda x_{1 .. n} . \phi(x_1) \odot \psi(x_1) \odot \ldots \odot \psi(x_n)
\]

(9) \[
\lambda \phi \in \text{Set} \ \forall \phi \odot \psi \odot \ldots \odot \psi \odot \phi
\]

(10a) \[
\text{sleep or laugh} = \lambda x_{e < t} . ||\text{sleep or laugh}|| (x) = 1 \lor ||\text{laugh}|| (x) = 1
\]

(10b) \[
\text{buy or sell} = \lambda x_{e < t} . ||\text{buy or sell}|| (x) = 1 \lor ||\text{sell}|| (x) = 1
\]

3 Wh-questions

3.1 Wh-words as indefinites

• Cross-linguistic motivation
• Karttunen’s denotations for wh-phrases

(11a) \[
||?A|| (w_0) = \lambda w . \lambda \phi_{< s, < s, t >} . ||?A|| (w_0) = 1 \land \phi = \lambda v . (||\text{came}|| (v) (x) = 1)
\]

(11b) \[
||?A|| (w_0) = \lambda w . \lambda \phi_{< s, < s, t >} . ||?A|| (w_0) = 1 \lor ||?B|| (w_0)
\]

(12) \[
||?A|| = \lambda w . ||?A|| (w) = 1 \land \phi = \lambda v . (||\text{came}|| (v) (x) = 1)
\]

(13) \[
||?A|| (w_0) = \lambda w . ||?A|| (w_0) = 1 \lor ||?B|| (w_0) = 1 \land \phi = \lambda v . (||\text{came}|| (v) (x) = 1)
\]

3 Wh-questions

3.1 Wh-words as indefinites

• Cross-linguistic motivation
• Karttunen’s denotations for wh-phrases

(14) \[
||\text{Which linguists came}|| (w_0) = \lambda w . \lambda \phi_{< s, < s, t >} . \phi (w_0) = 1 \land \forall x . (||\text{linguists}|| (w_0)) (x) = 1
\]

Equivalently:

(14b) \[
||\text{Which linguists came}|| (w_0) = \lambda w . \lambda \phi_{< s, < s, t >} . \phi (w_0) = 1 \land \exists x . (||\text{linguists}|| (w_0)) (x) = 1
\]

Going from sets to lambdas:

(14c) \[
||\text{Which linguists came}|| (w_0) = \lambda w . \lambda \phi_{< s, < s, t >} . \phi (w_0) = 1 \land \exists x . (||\text{linguists}|| (w_0)) (x) = 1
\]

(14d) \[
||\text{Which linguists came}|| (w_0) = \lambda w . \lambda \phi_{< s, < s, t >} . \phi (w_0) = 1 \land \exists x . (||\text{linguists}|| (w_0)) (x) = 1
\]
Compositional Derivation

(15) a. Which linguists came?
b. [Which linguists] [?] t1 came]

(16) \[Which linguists\] = \[\lambda w_s \lambda x = e \lambda s.t. (x \in (\text{linguists})(w)) \land (Z(x)(\phi) = 1)\]

(17) a. \[t1 came\] = \[\lambda w_s \lambda x = e \lambda \phi \lambda s.t. (x \in (\text{linguists})(w)) \land (\phi = \text{That x came} \land \phi(w) = 1)\]
   
   For short: ‘That g(1) came’
b. \[? \{t1 came\} = \lambda w_s \lambda x = e \lambda \phi \lambda s.t. (x \in (\text{linguists})(w)) \land (\phi = \text{That x came} \land \phi(w) = 1)\]
c. \[\lambda t_1 \{t1 came\} = \lambda w_s \lambda x = e \lambda \phi \lambda s.t. (x \in (\text{linguists})(w)) \land (\phi = \text{That x came} \land \phi(w) = 1)\]
   
   – Wh-words as Indefinites

(18) a. \[Which linguists\] = \[\lambda w_s \lambda x = e \lambda s.t. (x \in (\text{linguists})(w)) \land (Z(x)(\phi) = 1)\]
b. \[A \{linguist\} = \lambda P \lambda x = e \lambda y = e \lambda \phi (y \in (\text{linguists})(w)) \land (P(y)(w) = 1)\]
c. \[A \{linguist\} = \lambda P \lambda x = e \lambda y = e \lambda \phi (y \in (\text{linguists})(w)) \land (P(y)(w) = 1)\]
   
   – What about ‘a linguist’ in object position?

(19) a. Jack [met a linguist]
b. [Meet(a linguist)](Jack)
c. \[a \{linguist\} = \lambda R \lambda x = e \lambda y = e \lambda \phi (y \in (\text{linguists})(w)) \land (R(y)(w) = 1)\]
   
   – A single entry for indefinites and wh-phrases: using Flexible Types.

(20) Let X be a type which ‘ends in t’, i.e. of the from \(<\tau_1, <\tau_2, <... <\tau_n, t >>> \>. Then the following is a possible denotation for ‘a linguist’ or ‘which linguist’:

a. Type: (a/which linguists) \(<\text{linguist}\) >
b. \[a/\{which linguist\} = \lambda x = e \lambda \phi (x \in (\text{linguists})(w)) \land (\phi = \text{That x came} \land \phi(w) = 1)\]