A semantics for degree questions based on intervals - negative islands and their obviation
Based on joint work with Marta Abrusan.

0. Intro: negative islands and their obviation

- GOAL: provide a new account for negative islands in degree questions:

(1) How fast is Jack driving
(2) *How fast isn’t Jack driving?

….and their obviation in certain cases (Fox & Hackl 2006)

(3) How fast are we not allowed to drive on this highway?
(4) *How fast are we allowed not to drive on this highway?

(5) ? How many children are you sure that Peter does not have?
(6) *How many children are you not sure that Peter has?

- Fox & Hackl’s generalization:

Negative islands get obviated if negation immediately scopes over a possibility modal or if scopes immediately below a necessity modal.

- Core ideas to be developed

  + Interval-based semantics: degree predicates may denote relations between individuals and intervals of degrees, rather than between individuals and degrees.

(7) How fast is Jack driving?
   Standard view : for what degrees d of speed, is Jack driving d-fast?
   Present Proposal: for what intervals I, is Jack driving at a speed included in I

   + As in Fox & Hackl (2006): A question is unacceptable if it can never have a maximally informative answer (cf. Dayal 1996). This will be the case in cases like (2), but not in cases like (3) and (5)

1. 1st attempt: an ‘exact’ meaning for the degree-variable

1.1. Meaning of degree-adjectives

(8) The table is 2-feet tall
   >> The table’s height is 2-feet
\[
[[\text{tall}]] = \lambda d. \lambda x \ (x\text{’s height} = d)
\]

1.2. Meaning of degree-questions

(9) a. How tall is the table?
    b. How\(_d\) [the table is d-tall]?

(10) For what d, the table’s height = d

Karttunen: in a given world \(w\), a question denotes the set of all its true elementary answers.

\[\rightarrow\] The denotation of (9) is necessarily a singleton set, containing the proposition that the table is d-tall, where d is the height of the table.

1.3. Negative islands

(11) How tall isn’t the table?
(12) For what d, the table’s height \(\neq d\)

>> infinitely-many answers that do not entail each other, hence bad, due to Dayal (1996)’s assumption

1.4. Dayal’s assumption

A question presupposes that it has a complete answer, i.e. a true answer that entails all the other true answers.

>> Derives the uniqueness presupposition of singular which-questions

Singular which-questions

(13) a. Which book did Peter read?
    b. For which individual books \(x\), Peter read \(x\)?

Denotation of (13) in a world in which Peter read B1 and B2 (Karttunen style Semantics)

(14) \{The proposition that Peter read B1, The proposition that Peter read B2\}

(13)’s presupposition is not met. It can only be met if Peter read exactly one book.

Plural which-questions

(15) a. Which books did Peter read?
    b. For which pluralities of books \(X\), Peter read \(X\)?
Same situation: because *which books* now binds a variable rather than ranges over every single book and every plurality of books, the denotation of (15), in the above situation, is:

(16) \{\text{that Peter read B1, that Peter read B2, that Peter read B1+B2}\}

(17) \{\text{that Peter read B1, that Peter read B2, that Peter read B1+B2}\}

The proposition that Peter read B1+B2 entails all the other true answers. Hence (15) is felicitous and is so as soon Peter read at least one book.

1.5. Accounting for obviation

F & H’s obviation facts appear to be predicted:

(18) How fast are we not allowed to drive on this highway?
    For what d, it is not allowed that our speed be exactly d?

There might be a unique true answer, just in case there is a single given speed such that we are not allowed to drive at that speed (but are allowed to drive slower and faster).

But….this is clearly not the reading we get here

Abrusan’s (2007) answer (reformulated in BS’s terms for this particular example): suppose that on this highway we are not allowed to drive faster than 75mph. Then the set of all the true answers is:

(19) \{\text{We are not allowed to drive at exactly 76 mph, we are not allowed to drive at exactly 77 mph,…..}\}

None of these propositions entails each other *logically*. Yet given world knowledge, it is actually the case that *we are not allowed to drive at exactly 76 mph* often contextually entails *we are not allowed to drive at exactly 77 mph* (because we expect that the law determine a *maximal* speed). Hence if we define *most informative answer* in terms of contextual entailment, we are fine. Abrusan (2007) goes further than that, and suggests that we could give up entailment as a primitive entirely, and talk in terms of the *maximally relevant, or noteworthy*, proposition (I won’t go through the examples). Dayal’s condition would then become: any question presupposes that it has a *maximally relevant answer*.

Note furthermore that, as observed in Spector (2004), the *exact* reading is sometimes clearly present:

(20) a. How many people are you sure Mary did not invite ?
    b. I am sure that she did not invite exactly 13 people, because she is so superstitious.
Note that on this reading (20) has a quite surprising presupposition: that there is a unique number n such that I am sure that Mary did not invite n people.

1.6. Problems

(21) How fast are we required to drive on this highway?

Suppose that it is required that our speed be between 45mph and 75mph

Set of true answers: Ø (There is no speed such that we are required to drive at exactly that speed)

1.7. Conclusion

- The exact reading exists and is able to predict the modal obviation facts
- It cannot be the only reading
- The other reading(s) have to be such that a) negative islands cases are bad under all the possible readings, and b) the examples where negative islands are obviated can be felicitous under (some of) these readings as well (since the exact reading is not the only reading for these examples – it is in fact a marginal reading)

2. A maximality based account

2.1. A monotonic semantics for degree adjectives

An exactly-based semantics does not appear to work.

“Standard” assumptions for degree-predicates:

- \[[tall]\] = \(\lambda d.\lambda x. x \text{ is } d\)-tall
- For any x, d, d’ d’ ≤ d: tall(d)(x) ⇒ tall(d’)(x)

Jack is six-feet tall ⇔ Jack is at least two-feet tall

2.2. Rullman’s (1995) maximality condition

(22) How tall is Jack?
(23) a. For what d: d is the maximal degree such that Jack is d-tall?
   b. For what d: Jack is at least d-tall and for no d’>d, Jack is at least d’-tall

(24) How tall isn’t Jack
(25) a. For what d: d is the maximal degree such that Jack is not d-tall
   b. For what d: Jack isn’t at least d-tall and for no d’>d, Jack is not at least d’-tall
   c. For what d: Jack is less than d-tall and for every d’>d, Jack is at least d’-tall
There is no such degree
NB: Rullman’s original proposal is more general than this; it also applies to which-
questions. Furthermore, Rullman actually used an exactly-based semantics.

Predicted Generalization: predicates of degree P that license an inference from P(d) to
P(d’), with d < d’ (upward-scalar predicates\(^1\)), should be incompatible with degree-
questions, i.e. questions of the form “For what d P(d)?” should be unacceptable with
such a P.

2.3…..Wrong predictions

Beck & Rullman (1999):

(26) How tall is it sufficient to be (in order to play basketball)?

Suppose it is necessary and sufficient to be 7-feet tall. Then it is a fortiori sufficient to be
8-feet tall\(^2\). Hence \(\lambda d. \text{it is sufficient to be } d\text{-tall} \) is, in first approximation, upward-scalar,
and yet (26) is felicitous, but is predicted to be unacceptable.

<Interestingly, according to Rett (2006), a subclass of Romanian degree-questions
happens to behave (more or less) as predicted by the maximality account>

Fox & Hackl (2005):

(27) How fast are we not allowed to drive?

\(\lambda d. \text{we are not allowed to be } d\text{-fast} \) is upward-scalar. Yet (27) is felicitous.

3. Replacing Maximal with Maximal Informativity

3.1. A Generalization

(28) How \(d \phi(d)?\)

- If \(\phi\) is downward-scalar (i.e. \(\phi(d+\epsilon)\) entails \(\phi(d)\)), the question asks for the highest
degree \(d\) such that \(\phi(d)\) is true
- If \(\phi\) is upward-scalar (i.e. \(\phi(d)\) entails \(\phi(d+\epsilon)\)), the question asks for the smallest degree
\(d\) such that \(\phi(d)\) is true.

3.2. Informativity and Dayal’s constraint

\(^1\) Counter-intuitively, upward-scalar predicates happen to be downward-monotonic relatively to their
individual, non-degree, argument: this is because with \(d<d’\), it turns out that the denotation of \(d\text{-tall} \) includes that denotation of \(d’\text{-tall} \), not the reverse, since being at-least \(d’\text{-tall} \) entails being at least \(d\text{-tall} \).

\(^2\) First approximation: \(\text{It is sufficient } S \text{ in order } T \iff \text{if } S, \text{ then } T \) – that’s the ‘mathematical’ interpretation
of the notion of ‘\(S\) is sufficient condition for \(T\)’; probably not correct for natural language.
Follows directly from the following general principle, which is likely to be “pragmatic” in some sense:

- A question asks for the most informative true answer, i.e. a true answer that entails all the other true answers.

We add:
- …and which is not already known to be the most informative true answer.

- Dayal’s assumption: we may predict a question to be infelicitous in a given situation if it is known in this situation that there is no most informative answer. In particular, if there simply cannot be a most informative true answer, the question will be always infelicitous, hence unacceptable$^3$.

But: now we have no account for why negation should intervene:

(29)  
   a. How tall isn’t Jack?  
   b. For what d, Jack isn’t d-tall  
   c. For what d, Jack is less than d-tall

Suppose that Jack’s height is just below 6-feet. Then the set of true answers is:

(30)  
   \{\text{Jack isn’t 6-feet tall, Jack isn’t 6 ½-tall, Jack isn’t 7-feet tall,\ldots}\}

Clearly $\text{Jack isn’t 6-feet tall}$ is the most informative answer.


Szabolcsi & Zwart (1993) proposes the following account of negative islands in general:

(31)  
   *?x […….NEG …… x] if the variable x ranges over a domain which is not closed under \text{complementation}

Illustration:

(32)  
   Which books didn’t you read?

Let $X$ be the maximal plurality of books that you read. Then the complement of $X$ is the plurality of books whose atomic members are all the books that you did not read, if there are any. The complement of $X$ will always be defined provided we assume that the

$^3$ No infelicity would ever be predicted by Dayal’s assumption if the set of “answers” were closed under (infinite) conjunction, since the conjunction of all true answers always entails all the true answers. Accounts of infelicity based on Dayal’s assumption are therefore not natural within theories in which the notion of complete answer is precisely defined as (a proposition that entails) the conjunction of all true answers, to the effect that the complete answer is always defined <Karttunen’s original proposal, Groenendijk & Stockhof>.
domain of individuals include the null object (S & Z assume that the domain of individuals is a Boolean algebra)

(33) How tall isn’t Jack?

S & Z: the domain of degrees is not closed under complementation.

>> Criticism: very stipulat[4]. Also, cannot account for F & H’s obviation facts.

5. F & H’s account: dense scales

5.1. Predicting negative islands

(34) How tall isn’t Jack?

Suppose Jack’s height is 180 cm (Jack’s height might have been measured in any country except in the US or in Britain!). The set of all true propositions of the form Jack is not d-tall is the following:
{…, Jack is not 180,000001 cm –tall, …, Jack is not 180,05 cm-tall, …, Jack is not 181 cm-tall, …,}

It will be apparent that there is no minimal degree d such that Jack is not d-tall. This is simply because for any d > 180cm, there is a d’ such that d > d’ > 180cm[5]. So Dayal’s condition is not met.

F & H must extend this account even to cases where the domain of degrees is not “intuitively” dense, such as cardinality measures (a certain sort of degrees) as in:

(35) *How many children doesn’t Jack have?

Suppose Jack has exactly 3 children. Then he does not have 4 children, but he also does not have 3.5 children, or 3.00001 children…

Crucially, if all scales are dense, degree predicates such as λd. ¬P(d), where P is a syntactically simple degree predicate (tall, d-many books, ...), always denote open intervals, hence have no minimal elements.

4 Note however that all the accounts currently entertained must make certain assumptions about the structure of the domain in which the degree-variable denotes. These assumptions are needed in order to ensure that the set of true answers is not always closed under conjunction, so as to be able to predict that sometimes Dayal’s assumption cannot be met. See in the connection fn.3 and 5.
5 The conjunction of the infinitely-many true propositions of the form Jack does not have d children obviously entails all the true answers. In fact this proposition can be expressed by means of a finite sentence, as it is the one expressed by Jack is no more than 180cm-tall. If this counted as an answer, the account would be lost.
5.2. Accounting for the modal obviation facts

(36) How fast are we not allowed to drive?

Suppose that the law states that our speed should be lower than 65 mph, and says nothing more. It follows that the set of worlds compatible with the law is \{w : our speed is lower than 65 mph\}. So for any speed \( v \) below 65 mph (however close to 65 mph), there is a permissible world in which our speed is \( v \). Hence for any speed lower than 65 mph, we are allowed to drive at that speed. On the other hand, we are not allowed to drive at 65 mph. Hence 65 mph is the lowest speed \( v \) such that we are not allowed to drive at speed \( v \). So Dayal’s condition can be met.

More generally, predicates of the form \( \lambda d. \neg \text{POSSIBLE}(P(d)) \) can denote closed intervals. Likewise for \( \lambda d. \text{NEC}(\neg P(d)) \).

5.3. Modularity and blindness to contextual information

(37) *How many children doesn’t Jack have?

A natural objection: even granting that it makes sense to say that Jack has 3.5 children, yet the exact number of children someone has is always an integer. So \textit{Jack does not have 3} \( + \varepsilon \) children is known to be equivalent to \textit{Jack does not have 4 children}, and there is in fact a true answer that entails all the other ones.

F & H have to assume a very strong modularity assumption: presumably, the knowledge that the number of children someone has is an integer is a form of lexical/encyclopedic knowledge. Importantly, though, this knowledge is not purely logical, given some reasonable notion of logicality (one that is blind to lexical semantics/encyclopedic knowledge). F & H’s central claim is that Dayal’s condition is computed only on the basis of the purely logical meaning of the question, i.e. is blind to contextual, encyclopedic or lexical information.

While F & H do provide some intriguing arguments for this view (some of which are completely independent of degree-questions), we want to investigate an alternative. We’ll see that even if F & H’s account were right, the ideas that we will have explored have to be maintained on independent grounds.
6. Our alternative: a semantics based on intervals

6.1. An interval-based semantics for comparatives
(Schwartzschild & Wilkinson 2002, Heim 2006)

6.1.1. Quantifiers in comparatives: a puzzle (Schwartzschild & Wilkinson)

- A toy-theory of comparatives

(38) a. Jack is taller than Mary is
b. The maximal degree d s.t. Jack is d-tall is above the maximal degree d’ s.t. Mary is d’-tall

(39) a. Jack is \([-\text{er} [\text{than}_d \text{ Mary is d-tall}] \text{ d-tall }]\]
(40) \([-\text{er} [\text{than}_d \text{ Mary is d-tall}] \lambda d'. (\text{Jack is d'-tall})\]

\begin{align*}
[[\text{than}_i \text{ Mary is i-tall}]] &= \text{MAX} \{d: \text{Mary is } d\text{-tall}\} \\
[[\text{than}_i]] &= \lambda d. \lambda D_{<d,t>}. \text{MAX}(D) > d \\
[[-\text{er}]] &= \lambda d. \lambda D_{<d,t>}. \text{MAX}(D) > [[\text{than}_i \text{ Mary is i-tall}]] \\
&= \lambda D. \text{MAX}(D) > \text{MAX} \{d: \text{Mary is } d\text{-tall}\} \\
&= \lambda D. \text{MAX}(D) > \text{Mary’s height}
\end{align*}

\begin{align*}
[[[-\text{er} [\text{than}_d \text{ Mary is d-tall}] \lambda d'. (\text{Jack is d'-tall})]] \\
&= \text{MAX} (\lambda d'. \text{Jack is d'-tall}) > \text{Mary’s height} \\
&= \text{Jack’s height} > \text{Mary’s height}
\end{align*}

- A problem with quantifiers (Schwartzschild & Wilkinson)

(41) Mary is taller than every student

\begin{align*}
[[\text{than}_i \text{ every student is i-tall}]] &= \text{MAX} \{d: \text{every student is at least } d\text{-tall}\} \\
&= \text{MAX} \{d: \text{the shortest student is at least } d\text{-tall}\} \\
&= \text{the shortest student’s height}
\end{align*}

So (41) is predicted to mean that Mary is taller than the shorter student. This is plainly wrong. What we want is: for every student x, Mary is taller than x.

- Modals

(42) Mary is driving faster than she has to

Predicted to mean: Mary’s speed is above the minimal permitted speed → ok
(43) Mary is driving faster than she should

Predicted to mean the same as above → incorrect

(43) is actually equivalent to:

(44) Mary is driving faster than she is allowed to

What we want: For every permissible world w, Mary drives faster in the actual world than in w.

(45) ? Mary solved more problems than the teacher had demanded that she did

Ambiguous?

French

(46) Marie est restée plus longtemps qu’elle (ne) devait
Marie stayed longer than she (so-called expletive ne) must-past

>> Ambiguous.

6.1.2. An interval-based semantics for comparatives: the intuition

• \([\text{tall}] = \lambda \mathbf{D}_{<d,t}. \lambda \mathbf{x}. \mathbf{x}'s \text{ height} \in \mathbf{D}\)

(47) Jack is taller than Mary is

\([\text{than}_0 \text{ Mary is D-tall}] = \text{the set of intervals } \mathbf{D} \text{ such that Mary's height} \in \mathbf{D}\)

Jack’s height is above the smallest interval that contains Mary’s height.

(48) Mary is taller than every student is

\([\text{than}_0 \text{ every student is D-tall}] = \text{the set of intervals } \mathbf{D} \text{ such that for every student } \mathbf{x}, \mathbf{x}'s \text{ height is in } \mathbf{D}\)

The smallest such interval is [d1 d2], with d1 the height of the shortest student and d2 the height of the tallest student. Hence (48) ends up meaning that Mary’s height is above d2.

6.2. An interval-based semantics for degree-questions: explaining the basic pattern

6.1.1. The proposal

• LF for \textit{How tall}:

(49) \texttt{How}_{D_{<d,t}} [\ldots\text{D-tall}\ldots\ldots]., \text{with the variable } \mathbf{D} \text{ ranging over intervals}
(50) a. How tall is Mary?
   b. For what interval I, Mary’s height is in I?

----------------------------------[---(---M’s height ---)---]----------------------------------

Most informative answer: take I = {Mary’s height}

6.2.2. Negative islands

- Negative islands:

(51) a. #How tall isn’t Mary?
   b. For what interval I, Mary’s height is not in I?

[0------(---------------)----------]---M’s height---[------(--------)----------]

Suppose Mary’s height is d (with d ≠ 0 – which we argue is a presupposed by the question). Then any interval I that does not include d is such that Mary’s height is not in I. The set of all such intervals is exactly the one that includes a) all the intervals contained in [0, d] (= I₁) and b) all the intervals contained in ]d, +∞) (= I₂). Now let I₃ be an interval contained in I₂ (for instance, take I₃ = I₂). Then the (true) proposition that Mary’s height is not in I₃ does not entail the proposition that Mary’s height is not in I₁. Likewise, for any I₄ included in I₁, the (true) proposition that Mary’s height is not in I₄ does not entail that Mary’s height is not in I₂. Hence there is no interval I such that the proposition that Mary’s height is not in I entails all the other true propositions of the same form, and Dayal’s condition cannot be met.

6.2.3. F & H’s obviation facts

(52) a. How fast are we not allowed to drive?
    b. For what I, it is not allowed that our speed be in I?

Suppose the law states that our speed must be at most 75 mph. Then any I that is entirely above 75pm is such that it is not allowed that our speed be in I. The strongest true propositions of this form is obtained by taking I = ]75mph, +∞). Dayal’s condition is met.

(Same for “How fast are we required not to drive”)

(53) a. # How fast are we allowed not to drive?
    b. For what I, it is allowed that our speed be not in I?

- FACT:
Not being allowed to drive at a speed in $I_1$ entails Not being allowed to drive at a speed in $I_2$ \iff $I_2$ is included in $I_1$

- **HENCE**: The most informative true answer, if it exists, is based on an interval that includes all the intervals that yield a true answer (call this interval the ‘most informative true interval’):

(i)  case a: there is no particular speed $d$ such that we must drive at exactly $d$.
- for any $d$, we are allowed not to drive at a speed included in $[d,d]$ 
- given (ii), the most informative true interval, if it exists, is $[0, +\infty[$ 
- but this is impossible: in every possible world, our speed is in $[0, +\infty[$

(ii) case b: our speed must be exactly $d$.
- the for any $d’$ distinct from $d$, we are allowed not to drive a speed included in $[d’,d’]$ 
- Unless $d = 0$, given (ii), the most informative true interval, if it exists, is $[0, +\infty[$, which is impossible

(iii) Dayal’s presupposition can be met only in the degenerate case where $d = 0$, but in this case the question is vacuous (and so there is no maximally informative answer in our sense)

### 6.3. Predicted reading

(54) How fast are we supposed to drive?

Suppose that one the highway we should drive between 45mph and 75mph. Then the complete answer is predicted to state exactly this. We’ll argue later that there is actually another reading. But right now let me consider two special cases:

Case a) there is a minimal speed \gg the complete answer should state what this minimal speed is.

Case b) there is a maximal speed \gg The complete answer should state what this maximal speed is.

The important point is that we can show that the interval-based reading really exists:

(55) a. Jack and Peter are devising the perfect Republic. They argue about speed limits on highways. Jack believes that people should be required to drive between 50mph and 70mph. Peter believes that they should be required to drive between 50mph and 80mph. Therefore…

b. Jack and Peter do not agree on how fast people should be required to drive on highways
Two problems:
- we predict cases that don’t exist
- we fail to predict the ‘standard’ reading:
As a consequence, how fast and how slow are predicted to yield equivalent questions.

7. The PI-operator

7.1. Back to Comparatives

7.1.1. The ambiguity problem

(56) Jack is driving faster than he has to/is required to
(57) Jack is driving faster than he should
(58) Jack is driving faster than he is supposed to

Suppose the minimal speed is 45mph and the maximal speed is 75mph.

Both sentences are predicted to mean that Jack’s speed is above 75mph. This is correct for (57) and (58), not for (56). (56) means that Jack’s speed is above 45mph, with no implication he is breaking the rule. Maybe this reading is also available for (57).

Can we replicate this with degree questions?

Request for judgments:

(59) Why are you arresting me? How fast should I drive?
(60) Why are you arresting me? How fast am I supposed to drive
(61) Why are you arresting me? How fast do I have to drive?

7.1.2. The PI-operator (S & W, Heim)

(62) \([\text{fast}_1]_{<d,\langle e,\rangle>} = \lambda d. \lambda x. x’s \text{ speed is at least } d\)
(63) \([\text{fast}_2]_{<\langle d,\rangle, <\langle e,\rangle>} = \lambda D_{<\langle d,\rangle, <\langle e,\rangle>} \lambda x. x’s \text{ speed is in } D\)

- The PI-operator (reversing the order of arguments – this is harmless)
  (point-to-interval)
(64) \(\Pi = \lambda P_{<\langle d,\rangle, <\langle e,\rangle>} \lambda I_{<\langle d,\rangle, <\langle e,\rangle>} \max(P) \in I\)

(65) \(\Pi[\lambda d. \text{Jack is at least d-fast}] = \lambda I. \max\{d: \text{Jack is at least d-fast}\} \in I = \lambda I. \text{Jack’s speed is in } I.\)

(66) \(\Pi[\text{fast}_1]_{<\langle d,\rangle, <\langle e,\rangle>} = \lambda D \lambda x. (\Pi[\lambda d. \text{fast}_1(d)(x)])(D)\)
The crucial point is that \(\Pi\) can in principle appear higher than just above the lexical degree predicate.
7.2. The proposal

(67) \[[\text{fast}]\rangle_{<d,t>} = \lambda d. \lambda x. x's speed is at least d

LF of degree questions:

(68) How fast is John?
(69) For what interval I, (\Pi (\lambda d. \text{John is d-fast}) (I))?

(70) How fast are we required to drive?

Two possible LFs:

(71) For what interval I, it is required that (\Pi (\lambda d. \text{we are d-fast}) (I))

= For what interval I, it is required that our speed be in I?

(72) For what interval I, \Pi (\lambda d. \text{it is required that we drive at least d-fast})?

'\Pi (\lambda d. \text{it is required that we drive at least d-fast})' denotes the set of intervals that include the maximal speed s such that we are required to drive at least s-fast. Suppose that on the highway we must drive between 45mph and 75mph; then this maximal speed is 45mph; and therefore '\Pi (\lambda d. \text{it is required that we drive at least d-fast})' denotes all the intervals that include 45mph. The most informative answer turns out to be the one you get by just taking I = \{45mph\}

>> This is the reading predicted by standard treatment

• Crucially, \Pi (\lambda d. \text{Jack didn't drive d-fast}) is not defined, because there is no maximum in '\lambda d. \text{Jack didn’t drive d-fast}'. So \Pi cannot scope above negation, and negative islands are still accounted for.

• What are the attested readings? Based on how things work in comparatives, we expect \Pi to scope preferable above require but below should and supposed to

(73) Why are you arresting me <driving clearly too fast>?
   a. # How fast was I required to drive?
   b. How fast should I have driven?
   c. How fast was I supposed to drive?

7.3. fast vs. slow

Note that with a high PI, how fast is not equivalent to how slow:

(74) How slow are we required to drive on this highway?

(75) For what interval I, \Pi (\lambda d. \text{it is required that we drive at least d-slow})?
Assumptions:
- being at least $d$-slow $\Leftrightarrow$ being at most $d$-fast
- The maximal degree of slowness is the minimal degree fastness.

‘$\Pi(\lambda d. \text{it is required that we drive at least } d\text{-slow})$’ denotes the set of intervals that include the maximal degree of slowness $d$ such that we are required to drive at least $d$-slow, i.e. the minimal speed such that we must be at most $d$-fast. In the above scenario, this minimal speed happened to be 75mph.

8. Further Predictions: Quasi-Negative Islands

(76) How many children does none of these ten women have?

Prediction: there is a number $n$, such that no woman has $n$ children or more, and for every $m$ below $n$, some woman has exactly $m$ children.

(77) How much money isn’t Jack required to give to anyone?

Our prediction: For some amount $a$, Jack isn’t required to give $a$ to anyone, and for every amount $a’$ below $a$, he has to give $a’$ to someone.

(78) a. Context: Jack borrowed money from various people. Now he has to reimburse everybody.
   b. # How much money isn’t Jack required to give back to anyone?

Consider now:
(79) a. One day, Jack borrowed one euro from a friend. The next day, he borrowed two euros from another friend. The day after, he borrowed three euros from yet another friend…and so on: on every day, he borrowed one more euro than on the previous day. I don’t know when all this started, but I know that today he has to reimburse everybody, so I wonder:
   b. How much money isn’t Jack required to give back to anyone?

9. Truncation

See the paper.

Appendix: A remaining problem?

(80) How fast are we not allowed to drive on this highway?

D. Fox’s observation (p.c.): In this case the PI-operator can only scope $low$ (cannot scope over negation). So the only predicted reading is:
For what interval I, our speed should not be in I?

Suppose that in fact our speed should be in [45mph 75mph]. Then all the intervals I such that our speed should not be in I are those below 45mph and those 75mph. As a result there is no more informative answer. Hence the question is infelicitous – does not seem right.

>>Prediction: this question presupposes that
   a) there is a maximal speed and no minimal speed OR
   b) there is a minimal speed and no maximal speed OR
   c) our speed must be below x OR above y.

But b) seems to be excluded by the question….

Fox’s conclusion (p.c.): the PI-operator should in fact be defined in terms of maximal informativity, not maximality in the absolute sense. But then we lose the account of negative islands….Density would still be needed.

A repair:

*How fast* presupposes that the maximally informative interval is not of the sort [0, x] or [0, x[.

But…what are the facts?

<Talking about a swimming pool which is forbidden to very young children>

How old can’t a child be in the swimming pool?

**SELECTED REFERENCES**


Rett, J. 2007. Antonymy and Evaluativity, SALT 17


