SCALAR IMPLICATURES : LOCAL OR GLOBAL ?
EXHAUSTIVITY AND GRICEAN REASONING*

Two current ‘globalist’ accounts of scalar implicatures that address (some of) Chierchia’s puzzles:

a) Sauerland (2001): neo-gricean account based on the notion of scalar alternatives, and treating SIs as ‘generalized’, not context-dependent (or if they are, the procedure that computes them isn’t. The selection of the ‘strengthened’ reading may well be context-dependent)

b) Van Rooy (2002): scalar implicatures are a sub-case of exhaustive readings of answers. The procedure that generates them is essentially context-dependent, as it makes explicit reference to a question under discussion.

In this talk, I aim at unifying both approaches by showing that the exhaustivity facts (including scalar implicatures) can be derived from the gricean reasoning, in a way that is reminiscent of Sauerland’s ideas.

A) Local effects and imperfections of standard neo-gricean accounts

According to neo-gricean accounts, scalar implicatures are computed as follows: given a sentence S containing a scalar term t, S is to be compared to all sentences which can be obtained from S by replacing t with a term belonging to t’s scale. For any such scalar alternative S’ such that S’ asymmetrically entails S, the hearer infers that S’ is not part of the speaker’s beliefs. (hereafter, rule R1; this derives the so-called epistemic implicatures). The underlying principle motivating this inference is Grice’s first maxim of Quantity. Assuming further that the speaker is maximally informed, the hearer infers that S’ is in fact false according to the speaker (hereafter, rule R2).

(0) A or B
(1) A and B
(2) (A or B) but not (A and B)

Reasoning from (0) to (2):

a) if the speaker had believed (1) he wouldn’t have uttered (0), which is weaker.
b) Hence it is not the case that the speaker believes (1) to be true

* Many thanks to Gennaro Chierchia, Paul Dekker, Paul Egré, François Récanati, Philippe Schlenker, Dan Sperber, Robert van Roooy, Balder ten Caten, Henk Zeevat, to the audiences of the Dip Colloquium of Amsterdam and of the Philosophy of Language Workshop of UCLA for helpful discussions and encouragements.
\neg K(A \land B)
c) The speaker is relatively well informed
d) The speaker believes that (1) is false
K\neg(A \land B)

Direct implicatures are predicted to show up in UE contexts and to disappear in non-increasing contexts, e.g. DE and non-monotonic contexts.
Indirect implicatures are predicted to appear in DE contexts and to disappear in non-decreasing contexts, e.g. UE and non-monotonic contexts:
(3) Peter doubts that both Peter and Mary will come
(3’) Peter believes that Peter or Mary will come

1) Problems when a scalar term is under the scope of an operator (Chierchia (2001))

A) Modalities and Attitude verbs

(3) Peter believes that some of the students are waiting for him
According to Chierchia, (3) can implicate:
(4) Peter believes that some but not all of the students are waiting for him
(3) should only implicate (4’):
(4’) It is not part of Peter’s beliefs that all the students are waiting for him

BUT:

(5) It is certain that some of the students are waiting for Paul
does NOT implicate (6)
(6) It is certain that some but not all of the students are waiting for Paul
(5) DOES implicate (6’):
(6’) It is not certain that all the students are waiting for Paul.
For attitude verbs, local effects can be achieved by making reference to context of speech in their lexical entries:
X believes that P: X is in a context in which he is ready to assent to P
(There exists independent motivation for such a view – think of the problem of logical omniscience)
(7) John knows who came to the party → exhaustive reading
(8) John knows where there is a gas station → mention-some
X knows ? P = X’s knowledge enables him to satisfy completely the request for information expressed by ? P in the context of utterance.

B) Scalar terms under the scope of another scalar term

(9) (A or B) or C
(9) implicates (10)
(10) (A or B) or C and (not(A and B) and not(A and C) and not (B and C))
(11) Not ((A and B) and C)
The neo-gricean procedure predicts (9) should implicate (11), (12) and (13). But (12) entails that C is false, which is certainly not an implicature of (9). Let me call this problem that of UNWANTED NEGATIONS. And (11) and (13) taken together do not entail (10).

(14) Peter will eat some pears or all the apples
(14) implicates, among others:
(15) Peter will not eat all the pears
Negations of scalar alternatives:
(16) Peter won’t eat some pears and all the apples
(17) Peter won’t eat all the pears and all the apples
(18) It is not the case that Peter will eat all the pears or all the apples
(16) and (17) are indeed implicated by (14), but (15) cannot be derived from them.
(18) is certainly not implicated by (14), since it entails that Peter will NOT eat all the apples, while (14) obviously implicates that maybe he will.

C) Scalar terms under the scope of a universal quantifier

(19) Each of the students read Othello or King Lear
(20) Not all students studied both Othello or King Lear
(21) Each of the students studied Othello or King Lear but not both

Chierchia (2001) claims that (19) implicates (21), while only (20) is predicted on the neo-gricean account.

D) Non-monotonic contexts.

Neo-gricean accounts predict direct scalar implicatures disappear both in DE contexts and non-monotonic contexts while Chierchia claims that they disappear in DE contexts but not necessarily in non-monotonic contexts.

(22) Exactly three students studied Maths or Physics
According to Chierchia, (22) does implicate in some contexts (23):
(23) Exactly three students studied Maths or Physics but not both.

If true, (22) could be used in a situation where exactly three students studied Maths or Physics but not both, while a fourth one studied both, in which case (22) on its literal reading is false. The “strengthened meaning” is not logically stronger (nor weaker) than the “plain meaning”.

Non-monotonic contexts are the crucial contexts that should be able to decide between localism and globalism, since no globalist procedure is going to predict an implicature that is not stronger than the plain one (though a slight modification of Chierchia’s theory would make the prediction that direct SIs disappear in non-UE contexts).
(24) a. # Between two and five students read some of the books, and much more students read all the books
  b. ? Between two and five students read three books, and much more read five books (focus on “three” and “five” ?)
  c. ?# Between two and five students read this or that, and many other students read both this and that.

Maybe (22) does in fact implicate (24’)b in some specific contexts:
(24’) Exactly three students studied Maths or Physics but not both, and no student studied both.
(This is logically stronger than the plain reading)

II) Chierchia’s solution (see also Landman 2000)

Apply the neo-gricean procedure not to the full sentence, but to its smallest propositional constituent. Enter the result into the semantic composition procedure, and do the same for the next propositional constituent, except if you meet a DE operator. If a certain propositional constituent appears to belong to a (locally) DE context, cancel what you have done and apply the neo-gricean procedure to the full constituent, etc.

III) Sauerland (2001)

> Expand the set of alternatives
> Have more plausible inference rules
Ex :
(A or B) or C
Alternatives : A, B, C, ( A c B), (A c C), (B c C), (A c B) c C
(where ‘c’ is either ‘and’ or ‘or’)

Reasoning :
1) Primary implicatures :
a. –maxim of quality: K (A or B) or C
   - maxim of quantity: ¬K A  ¬KB  ¬KC    ¬K(A or B)  ¬K (A and B) ¬K(A or C) ¬K (A or C)
   ¬K (B or C)  ¬K (B and C)  ¬K((A and B) or C)  ¬K ((A and B) and C)
   ¬K((A or B) and C)

b. Derive all the logical consequences of a.
   For instance : ¬K ¬C
   Indeed if K ¬C, since K (A or B) or C, then K ( A or B), which is contradictory….
   So we have ¬K ¬A          ¬K ¬B              ¬K ¬C
   Intuition : C cannot be known to be false, since in that case (A or B) would have been a better utterance that (A or B) or C

2) Secondary implicatures:
If \( \neg K P \) is a primary implicature and \( \neg (K \neg P) \) is not, then derive \( K \neg P \).

Ex. From \( \neg K(A \text{ and } B) \), derive \( K \neg(A \text{ and } B) \)...identically for \( (B \text{ and } C) \), \( (A \text{ and } C) \)

>>>only one of the disjuncts is true

**IV) Van Rooy (2002)'s solution: exhaustivity**

Scalar implicatures are context-dependent, and depend on the question under discussion. For any sentence, find the QUD it is an answer to, and apply Exhaustification, e.g “maximize relevance” in a sense made precise. This derives (some of) Chierchia’s local effects.

If a certain question \( Q \) is under discussion and a certain sentence \( S \) is given as an answer to \( Q \), \( S \) is generally interpreted as "exhaustive".

The exhaustivity operator (Groenendijk & Stokhof 1984) operates on answers of the form 'GQ P', where GQ stands for a generalized quantifier and P for a predicate. The question under discussion is understood as "for which objects is P true of these objects ?".

The exhaustivity (\( \text{exh} \)) operator works as follows:

\[
\langle \text{exh} (GQ P) \rangle = 1 \text{ iff } \langle P \rangle \in (\text{Min }[GQ]), \text{ where } (\text{Min }[GQ]) \text{ is the set that includes only the minimal members of } [GQ], \text{ i.e:}
\]

\[
\text{Min }[GQ] = \{ x \mid x \in [GQ] \text{ and there is no } x' \text{ in } [GQ] \text{ such that } x' \subset x \}
\]

(\( \subset \) = “is a proper subset of”)

**Example:**

(25) a. Among John, Mary and Peter, who came?

b. John or Mary came

\[
\langle [\text{John or Mary}] \rangle = \{ \{J, M, P\}, \{J, M\}, \{J, P\}, \{J\}, \{M, P\}, \{M\} \}
\]

(\( \text{Min }[\text{John or Mary}] \)) = \{ \{J\}, \{M\} \}

\[
\langle \text{exh} (\text{John or Mary came}) \rangle = 1 \text{ iff } \langle \text{came} \rangle \in \{ \{J\}, \{M\} \} \text{ i.e. iff only John came or only Mary came.}
\]

Van Rooy shows that when exhaustification is applied to monotone increasing contexts, it can solve some of Chierchia's puzzles.

However, if exhaustification is applied to a sentence 'GQ P' where GQ is decreasing, exhaustification as defined above leads to unrealistic implicatures: "less than two chemists came" should implicate that nobody came! So Van Rooy uses a second exhaustivity operator (\( \text{exh}' \)) in these cases, following Stechow & Zimmermann (1984):

\[
\langle \text{exh}' (GQ P) \rangle = 1 \text{ iff } \langle P \rangle \in (\text{Max }[GQ]), \text{ where } (\text{Max }[GQ]) \text{ is the set that includes only the maximal members of } [GQ]
\]

1 I reformulate Groenendijk & Stockhof’s exhaustivity operator in more simple terms, but the difference is immaterial.
There are several problems with this account. First, the second rule of exhaustification makes wrong predictions:

(26) a. Among the chemists and the philosophers, who came?
   b. Less than two of the chemists

Exhaustification leads to b':

b' . Exactly one chemist and all the philosophers came.

But b. does not seem to implicate b'; b. actually does indeed suggest that some chemist came, but does not implicate anything regarding non-chemists. It rather suggests that the speaker does not know much about them.

Second, these two rules are unable to account for cases where the speaker combines increasing and decreasing quantifiers, thus creating a non-monotonic GQ, as in (27b):

(27) a. Among the chemists, the philosophers and the linguists, who came?
   b. Less than two chemists but one philosopher came

If we apply the first exhaustivity operator, what we get is that b. implicates that no chemist and no linguist came, while exactly one philosopher came.

If we apply the second exhaustivity operator, what we get is that exactly one chemist, all the philosophers and all the linguists came.

None of these predictions is in fact borne out. Rather, it seems that (27b) implicates that at least one chemist came, exactly one philosopher came, and that the speaker does not know much about linguists.

V) Context-dependency

(28) Where is there a gas station ?
   In this street or in that street

(29) Who saw this movie among this group of students ?
   Peter or Mary

>>It seems that there is a correlation between exhaustive readings and scalar implicatures.

(30) Do you have time for a drink ?
   Some of my students are waiting for me

(31) Which of your students read which plays by Shakespeare ?
   Each of them read King Lear or Othello.

(32) Among linguists and philosophers, who read which plays by Shakespeare ?
   Between two and five linguists read King Lear or Othello.
Between two and five linguists read *King Lear* or *Othello* but not both, and nobody else read any play by Shakespeare.

**VI)** “Generalized exhaustivity effects”

(33) a. Why is John in a bad mood?
   b. John met Peter, or Jack and Mary

Not only does (33)b. implicate an exclusive reading for “or”. It is also interpreted quite naturally as excluding a situation in which John met Peter and Jack but not Mary, a situation that would make b. true under an exclusive construal for “or”. In fact, b. implicates that “Either John met Peter and neither Jack nor Mary, or he met Jack and Mary but not Peter”, i.e. “Among Peter, Jack and Mary, John met only Peter, or John met only Jack and Mary”.

**VII)** Goal of this paper: deriving exhaustivity

In the next sections, I show that both scalar implicatures and exhaustification of answers can be understood as the outcome of a pragmatic reasoning that is based on the gricean maxims. I will first offer a precise formalization of the gricean reasoning, meant to replace the two rules R1 and R2. I will then show that it is possible to predict the facts reviewed above by defining carefully what counts as an "alternative answer" for a given answer to a certain question under discussion. In a way, my proposal aims at unifying “globalist” gricean accounts (Sauerland) and accounts based on exhaustivity (Van Rooy), by showing that exhaustivity can be derived from gricean maxims.

**B) Formalizing the gricean reasoning**

**I) Relevance, Information states and relevant information**

Relevance is defined as “answerhood”. Being relevant is simply being a possible answer to the question under discussion.

A Partition Semantics for questions:

A question divides the logical space into disjoint cells, i.e. induces a partition on the set of worlds. Or, put differently, it defines an equivalence relation over worlds. Two worlds w₁ and w₂ are equivalent for a question Q if they determine the same answer

Notation:

- \( w \ R_Q w' \) \quad w and w’ belongs to the same “cell”
- \( R_Q (v) = \{ w \mid w \ R_Q v \} \) \quad (= the set of worlds equivalent to v, or v’s cell)
- \( \alpha (w) \) \quad \alpha is true in w (alternatively : \( w \in \alpha \))
- \( \alpha \subseteq \beta \) \quad \alpha is a subset of \( \beta \); \( \alpha \) entails \( \beta \)
- \( \alpha \subset \beta \) \quad \alpha is a proper subset of \( \beta \); \( \alpha \) asymmetrically entails \( \beta \)
The proposition $\alpha$ expressed by A is supposed to meet the condition of strong relevance.

Def 1 (strong relevance): A proposition $\alpha$ (= set of worlds) is strongly relevant with respect to a question $Q$ if

a) $\exists w, (R_Q(w) \cap \alpha) = \emptyset$ (i.e.: $\alpha$ excludes at least one cell)

and

b) $\forall w, (\alpha(w) \leftrightarrow (R_Q(w) \subseteq \alpha))$ (i.e.: $\alpha$ does not distinguish between two worlds that belong to the same cell, i.e. provides no irrelevant information)

These conditions ensure that $\alpha$ excludes at least one cell, and that $\alpha$ does not convey any irrelevant information. For a proposition to be strongly relevant relatively to a question $Q$, it must be the case that $Q$ itself is not tautological, in the sense that it determines at least two cells.

Hereafter, I will simply use the word “relevant”, as I will not consider weakly relevant propositions.

The speaker’s information state is modeled as a set of worlds, i.e. a proposition. As an agent believes a lot of things that are irrelevant in the context of a given question, it is useful to define what counts as the relevant information contained in a certain information state:

Def 2 (relevant information): Let $i$ be an information state and $Q$ a question. Then we define $i$ relativized to $Q$, written as $i/Q$, as follows:

$$i/Q = \{w | \exists w', (w' R_Q w \text{ and } w' \in i)\} = \cup_{w \in i} R_Q(w).$$

Thus $i/Q$ is simply the union of all the cells that are not excluded by $i$.

II) Optimal answers among a set of propositions

The gricean reasoning is based on the idea that $\alpha$ (the proposition given as an answer) must be compared to a certain set of alternative propositions which the speaker could have chosen instead of $\alpha$. This alternative set, call it $S$, must contain $\alpha$ itself, and be such that all its members are relevant. The hearer's task when interpreting the speaker's utterance is to address the following question: given that the speaker has preferred $\alpha$ to all the other members of $S$, what does this entail regarding his information state $i_0$? First, the speaker must believe $\alpha$ to be true (Grice’s maxim of quality), i.e. $i_0$ must entail $\alpha$. Second, $\alpha$ must be optimal in the sense that there must be no more informative proposition in $S$ entailed by the speaker’s beliefs (Grice’s maxim

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2 As my formulation makes clear, I am now adopting the simplifying view that what the hearer compares are propositions.

3 The exact definition of alternative sets is the topic of section C.
of quantity), i.e. there must be no proposition \( \alpha' \) such that \( i_0 \) entails \( \alpha' \) and \( \alpha' \) asymmetrically entails \( \alpha \):

**Def 3 (optimal answer)**: Let \( S \) be a set of propositions, all relevant with respect to a certain question \( Q \). Then \( \alpha \) is an optimal answer in \( S \) with respect to \( Q \) in information state \( i \) if:

a) \( i/Q \subseteq \alpha \) (which is in fact equivalent to \( i \subseteq \alpha \), since \( i \subseteq i/Q \) and \( \alpha \) is relevant) and

b) For any \( \beta \) member of \( S \), if \( i/Q \subseteq \beta \), then it is not the case that \( \beta \subset \alpha \).

- Condition a) states that for \( \alpha \) to be an optimal answer in \( S \), it must be known by the agent, e.g. entailed by his information state
- Condition b) states that there is no proposition \( \beta \) in \( S \) which is known to be true by the agent and which asymmetrically entails \( \alpha \). (the intuition being that otherwise the proposition \( \beta \) in question would have been a better answer to \( Q \) than \( \alpha \). This is the formal counterpart of Grice's maxim of quantity)

Put differently, \( i_0 \) must belong to the following set \( I(S, \alpha, Q) \):

**Def 4**: \( I(S, \alpha, Q) = \{ i \mid \alpha \) is an optimal answer in \( S \) with respect to \( Q \) in information state \( i \}")

\[ = \{ i \mid i \subseteq \alpha \text{ and } \forall \alpha' (\alpha' \in S \text{ and } i \subseteq \alpha') \rightarrow \neg (\alpha' \subset \alpha) \} \]

So if a certain proposition \( \beta \) is entailed by no member of \( I(S, \alpha, Q) \), the hearer can conclude that \( \beta \) is not part of the speaker’s belief. This reasoning plays the role of rule R1. It is immediately predicted that if the speaker utters a sentence \( P \) of the form “A or B” and if the propositions expressed by A and by B belong to the alternative set \( S \), as I will assume (so does Sauerland (2001)), then the speaker cannot know A to be either true or false: if A were true, then A would have been a better answer than \( P \); if A were false, B would be true (since \( P \) is), and B would have been a better answer than \( P^4 \).

**III) “The speaker is (relatively) well informed”**

Gricean accounts all need a jump from “the speaker does not know that \( P \)” to “The speaker knows that not \( P \)”, and the implicit motivation for this is that the hearer considers the speaker to be “well informed”. Let the hearer therefore assume that the speaker is as informed as possible given the answer he made. This means that his information state \( i_0 \) is maximal in \( I(S, \alpha, Q) \) in the following sense: there is no \( i' \) in \( I(S, \alpha, Q) \) such that \( i' \) (relativized to \( Q \)) asymmetrically entails \( i_0 \) (relativized to \( Q \)). In other words, \( i_0 \) belongs to \( \text{Max}(S, \alpha, Q) \), defined as follows:

**Def 4**: \( \text{Max}(S, \alpha, Q) = \{ i \mid i \in I(S, \alpha, Q) \text{ and } \forall i' (i' \in I(S, \alpha, Q)) \rightarrow \neg (i'/Q \subset i/Q) \} \)

From this the hearer can conclude that if a proposition \( \beta \) is entailed by all the members of \( \text{Max}(S, \alpha, Q) \), then \( \beta \) is believed by the speaker. This reasoning plays the role of R2, but is not equivalent to it: there is no way of deriving an “unwanted negation”. In the

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4 Assuming that A and B are logically independent.
case of a disjunctive statement in which the disjuncts are logically independent, the
disjuncts and their negations are entailed by no member of I(S,α,Q), as shown above,
so that they cannot be entailed by any member of Max(S,α,Q) either, since
Max(S,α,Q) is included in I(S,α,Q).

From now on, whenever it clear what the question under discussion is, and considering
that the content of an alternative set only depends on the question under discussion
and the sentence uttered, I will simply write I(α) and Max(α) instead of I(S,α,Q) and
Max(S,α,Q). S(α) will denote the alternative set of α.

C) Alternative sets and Exhaustification

I. An example

Let P be of the form ‘(A or B) or C’, where A, B and C are logically independent.
Assume that P is uttered in a context in which A, B and C’s truth-values are what is
relevant i.e. as an answer to a question Q amounting to “Which sentence(s) are true
among A, B, and C?”

For any information state i, the relevant part of i in this context (i.e. i/Q) belongs to the
boolean closure of \{A,B,C\}. So we will loose nothing if we view information states as
sets of valuations of \{A, B, C\}, i.e. as propositions of the propositional language based
on \{A,B,C\}, where any such proposition actually stands for a class of propositions that
are all equivalent when relativized to Q. Let S(P) (the alternative set of P) be the
closure under union and intersection of \{A,B,C\}\(^5\). Intuitively, S(P) is the set of
positive answers to Q:

\[
S(P) = \{A,B,C,A\lor B,A\land B,A\land C,B\lor C,B\land C,(A\lor B)\lor C,(A\land B)\lor C,A\lor (B\land C) \ldots \}
\]

Assume i\(_0\) = ((A \lor B) \lor C)) \land (¬(A \land B) \land (¬(A \land C) \land (¬(B \land C)))). Then i\(_0\) \(\in\) I(P), since P is
the only – and therefore best - proposition in S(P) entailed by i\(_0\); i\(_0\) can also be
described as the set of the three following valuations:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>W2</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>W3</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

I now show that Max (P) = \{i\(_0\)\}, e.g. that i\(_0\) entails all the members of I(P). Suppose i\(_1\)
is an information state that is not entailed by i\(_0\) and that belongs to I(P). There is then
an element of i\(_0\) that does not belong to i\(_1\). Suppose W\(_1\) does not belong to i\(_1\). Then i\(_1\)
entails P': P' =  ¬(A \land (¬B \land ¬C)) = ¬A \lor (B \lor C)

But i\(_1\) belongs to I(P), and therefore entails P. Hence i\(_1\) also entails P'':

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\(^5\) As the reader will have noticed, I treat sentences both as sentences of the object-language and as names (in the
meta-language) of propositions, i.e. names of sets of worlds, in which case conjunction and disjunction are
understood as intersection and union.

\(^6\) I do not give the proof here, but it is in fact a subcase of a more general fact that is proved below.
P'' = ((A ∨ B) ∨ C) ∧ (¬A ∨ (B ∨ C)) = (B ∨ C)

But P'', which belongs to S(P), would have been a better answer than P in information state i₁, so that i₁ does not belong to I(P), contrary to the hypothesis. Things are similar if W₂ or W₃ does not belong to i₁ (by symmetry). Hence Max(P) = \{i₀\}, and P implicates i₀.

This proof can be generalized to all formulas whose only logical operators are disjunctions.

II. Background concepts

As shown in section A. IV, answers lead to different kinds of implicatures, especially regarding exhaustivity, depending on whether they are, intuitively speaking, positive or negative. But this cannot make sense so far, as I have not said precisely what it is for an answer to be “positive”. This is the goal of the present section.

I now assume that questions are all equivalent to something like:

Q: “For which x is P(x) true?”, where x is of any semantic type, and P is a certain predicate (simple or complex) that can be built in a natural language. I further assume that the domain of quantification is fixed and finite, and known to all participants. Thus any relevant answer to Q can be translated into the following propositional language L_Q: let (cᵢ)₀<ᵢ<n+₁ be an enumeration of names for each of the individuals of the domain. Then L_Q is the propositional language with disjunction and conjunction as its only binary connectors and based on the atomic sentences (Pᵢ)₀<ᵢ<n+₁, where Pᵢ translates P(cᵢ).

Now, relevant answers to Q can be seen as sets of valuations of (Pᵢ)₀<ᵢ<n+₁. And the relevant part of any information state can also be seen as a set of valuations. So we can assimilate information states to sets of valuations, without loosing anything.

Definitions:

1. Literal: a literal is an atomic sentence or the negation of an atomic sentence. A literal is positive if it is an atomic sentence, negative otherwise.

2. Sentence P favours literal L: a sentence or a proposition P favours a literal L iff there is a valuation V such that V(P) = V(L) = 1 and V_L(P) = 0, where V_L is defined as the valuation which is identical to V except for the value it assigns to L.

3. Sentence P essentially mentions literal L: A sentence P essentially mentions a literal L iff L occurs without a negation preceding it in every P' equivalent to P and such that the scope of all negations occurring in P' is an atomic sentence.

4. Positive sentence/positive proposition: a sentence or a proposition is positive (resp. negative) iff it favours at least one positive (resp. negative) literal and no negative (resp. positive) literal.

We can then prove the following theorems (Egré, Gliozzi & Spector):

Theorem 1: For any sentence P and any literal L, P favours L iff P essentially mentions L.
Theorem 2: A sentence $P$ is positive (resp. negative) iff $P$ is equivalent to a sentence which belongs to the closure of positive (resp. negative) literals under conjunction and disjunction.

**Corollary:** A sentence $P$ is positive iff it is equivalent to a sentence $P'$ which contains no negation.

We therefore have two characterizations of positive answers: an answer is positive if it is equivalent to a sentence which contains no negation, or, equivalently, if it favors at least one positive literal and no negative literal. This equivalence will prove helpful.

**III. The case of positive propositions: predicting exhaustification**

The alternative set of any positive proposition is defined as the set of all positive propositions.$^7$

**III.1. A first Example**

Consider the following dialogue:

(34) – Among John, Peter, Mary and Sue, who will come?
- Well, John will come, or Peter and Mary will come

I translate the answer into a propositional language containing four atomic sentences $A, B, C$ and $D$:

$P = A \lor (B \land C)$

$P$ quite clearly implicates $Q : Q = (A \land \neg B \land \neg C \land \neg D) \lor (B \land C \land \neg A \land \neg D)$ i.e. “either only John will come, or only Peter and Mary will”, which is exactly what exhaustification in Groenendijk & Stokhof”s sense would yield.

What I will now prove is that $\text{Max}(P) = \{Q\}$, from which it indeed follows that $P$ implicates $Q$.

First, I show that $Q \in I(P)$, i.e. $P$ is an optimal answer in $S(P)$ in information state $Q$. Suppose the speaker’s information state is $Q$. $Q$ can be represented as the following set of valuations, where a valuation is itself represented as the set of atomic sentences that this valuation makes true: $Q = \{\{A\},\{B, C\}\}$.

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<tr>
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<th>$A$</th>
<th>$B$, $C$</th>
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<td>$A, B$</td>
<td>$A, B, C$</td>
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<td>$A$</td>
<td>$A, C$</td>
<td>$B, C, D$</td>
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<td>$A$</td>
<td>$A, D$</td>
<td>$A, B, C, D$</td>
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<td>$A$</td>
<td>$A, B, C, D$</td>
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$^7$ It should be clear that the alternative set is dependent on the question under discussion, since “positivity” is defined in terms of the propositional language derived from the question under discussion via the translation procedure defined above.
By hypothesis, the speaker has to choose a proposition that belongs to the alternative set. This proposition must be entailed by Q and be such that there is no better proposition in the alternative set. Let Q’ be a positive sentence entailed by Q. Necessarily the valuation represented by \{A\} is in Q’. But then, the valuation \{A,B\} must be in Q’ too: if \{A,B\} were not in Q’, indeed, \neg B would be **favoured** by Q’, since there would be a valuation v making \neg B true in Q’ (namely v = \{A\}) and such that the valuation v’ identical to v except over B (v’ = \{A, B\}) would not be in Q’; so Q’ would favour a negative literal and not be positive, contrary to the hypothesis. By the same reasoning, \{A,C\}, \{A,D\} \{A,B,C\}, \{A,B,D\}, \{A,C,D\} and \{A,B,C,D\} must belong to Q’, and so does \{B,C,D\} (since \{B,C\} is in Q and therefore in Q’). So any positive proposition entailed by Q must include the following proposition, i.e. be entailed by it:

\{\{A\}, \{A,B\}, \{A,C\}, \{A,D\}, \{A,B,C\}, \{A,B,D\}, \{A,C,D\}, \{A,B,C,D\}, \{B,C\}, \{B,C,D\}\} \quad (= P)

But this set, which turns out to represent P, is a positive proposition which is entailed by Q and which entails all other positive propositions that are entailed by Q (as I have just shown). So P is the strongest positive proposition entailed by Q, i.e. Q \in I(P) (recall that I(P) is the set of all information states which make P an optimal answer among positive answers).

Second, I show that Max(P) = \{Q\}. This amounts to proving that Q entails all the members of I(P). Assume there is an information state i which belongs to I(P) and is not entailed by Q. Since i is not entailed by Q, then either \{A\} or \{B, C\} does not belong to i. Suppose \{A\} does not belong to i. On the other hand, i belongs to I(P) and therefore entails P. From which it follows that i entails P-\{A\}, i.e. i is included in the following set of valuations:

P - \{A\} = \{\{A,B\}, \{A,C\}, \{A,D\}, \{A,B,C\}, \{A,B,D\}, \{A,C,D\}, \{A,B,C,D\}, \{B,C\}, \{B,C,D\}\}. But this set is itself a positive proposition, since it can be checked that P-\{A\} favours no negative literal. In fact, P-\{A\} can be written as: (A\land (B\lor C\lor D))\lor (B\land C). So i entails a positive proposition that is stronger than P, namely P-\{A\}, which contradicts the hypothesis that i belongs to I(P). Things work similarly if \{B,C\} does not belong to i. Therefore there is no such i. From which it follows that Q entails all the members of I(P). Q.E.D

**III. 2. A second example**

**Context**: there are three professors and two students invited to a dinner

(35) - Who came among professors and students?

- two of the professors came

P₁ : professor-1 came  
S₁ : student-1 came

P₂ : professor-2 came  
S₂ : student-2 came

P₃ : professor-3 came
(35') \((P_1 \land P_2) \lor (P_1 \land P_3) \lor (P_1 \land P_3)\)

Let \(i_0\) be defined as:
\[
i_0 = ((P_1 \land P_2) \lor (P_1 \land P_3) \lor (P_2 \land P_3)) \land (\neg (P_1 \land P_2) \land \neg (S_1 \lor S_2))
\]

\(i_0\) : "exactly two professors came and no student came"

We show that \(\text{Max} ((35')) = \{i_0\}\), assuming that \(S(1')\) is the closure under union and disjunction of \(\{P_1, P_2, P_3, S_1, S_2\}\), e.g. the set of all positive propositions of the language whose only atomic sentences are \(P_1, P_2, P_3, S_1\) and \(S_2\).

I first prove that (35') is the strongest proposition in \(S(1')\) which is entailed by \(i_0\).

\(i_0\) can be represented as a set of valuations, and a valuation can be represented as a set of atomic sentences (those that are true in that valuation).

\[
i_0 = \{\{P_1, P_2\}, \{P_1, P_3\}, \{P_2, P_3\}\}
\]

Now, for \(P\) to be a best answer in \(S(35')\) for \(i_0\), \(P\) must be the smallest (strongest) positive proposition containing \(i_0\) if it exists. If \(P\) is positive and \(\{\alpha, \beta\}\) belongs to \(P\), then for any \(\gamma\) positive elementary proposition different from \(\alpha\) and from \(\beta\), \(\{\alpha, \beta, \gamma\}\) belongs to \(P\) too (otherwise \(P\) would favour \(\neg \gamma\), and \(P\) would not be positive). By the same reasoning, \(\{\alpha, \beta, \gamma, \delta\}\) also belongs to \(P\), for any positive elementary proposition \(\delta\) different from the previous ones, etc. All the sets of the type \(\{\alpha, \beta, \ldots\}\) belong to \(P\). So there is a smallest positive proposition \(P_0\) containing \(i_0\), which can be described as:

<table>
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<tr>
<th>P_0 :</th>
<th>P_1, P_2</th>
<th>P_1, P_3</th>
<th>P_2, P_3</th>
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<td>P_1, P_2, P_3</td>
<td>P_1, P_3</td>
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<td>P_1, P_2, P_3, S_1, S_2</td>
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<td>P_1, P_2, S_2</td>
<td>P_1, P_3, S_2</td>
<td>P_2, P_3, S_2</td>
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e.g. \(P_0 = (P_1 \land P_2) \lor (P_1 \land P_3) \lor (P_1 \land P_3) = (1')\).

I then show that \(i_0\) is such that it entails every \(i\) for which (35') is a best answer. Suppose \(i_1\) is not entailed by \(i_0\) but \(i_1\) entails (35'). Then \(i_1\) is such that either \(\{P_1, P_2\}\), \(\{P_1, P_3\}\) or \(\{P_2, P_3\}\) does not belong to \(i_1\). Suppose \(\{P_2, P_3\}\) does not belong to \(i_1\). Then \(i_1\) entails \((P_2 \land P_3) \land \neg P_1 \land \neg S_1 \land \neg S_2\), e.g. \((P_2 \land P_3) \lor (P_1 \lor S_1 \lor S_2)\).

But \(i_1\) also entails \((P_1 \land P_2) \lor (P_1 \land P_3) \lor (P_2 \land P_3)\), and therefore entails:
\[
(\neg (P_2 \land P_3) \lor (P_1 \lor S_1 \lor S_2)) \land ((P_1 \land P_2) \lor (P_1 \land P_3) \lor (P_2 \land P_3)),
\]

which in turn entails:
\[
((P_2 \land P_3) \land (P_1 \lor S_1 \lor S_2)) \lor (P_1 \land P_2) \lor (P_1 \land P_3)).
\]

And this last proposition is positive and asymmetrically entails \(P_0\) i.e. is better than \(P_0\) in information state \(i_1\). Hence \(i_1\) does not belong to \(I((35'))\), and (by symmetry) \(i_0\) is the
only maximal element of \( I((35')) \), e.g. \( \text{Max } ((35')) = \{i_0\} \). Therefore \((35')\) implicates \( i_0 \).

III. 3. Predicting exhaustification

In the general case, positive answers are predicted to be interpreted as exhaustive.

**Definitions:**

1. **Exhaustification:**

   Let \( P \) be any non-negative proposition, then the function \( \text{Exhaust} \) is defined as follows:
   \[
   \text{Exhaust}(P) = \{ V \mid V \in P \text{ and there is no valuation } V' \text{ in } P \text{ such that } V' \subseteq V \}\]

   This operator is the propositional counterpart of Groenendijk & Stockhof’s exhaustivity operator.

2. **Positive extension of a proposition** \( P \): for any non negative proposition \( P \), there is a unique positive proposition \( Q \) such that \( P \) entails \( Q \) and \( Q \) entails all the other positive propositions that \( P \) entails (i.e. \( Q \) is the strongest positive proposition that \( P \) entails). This can be shown by using the same reasoning as in the previous section: namely, you get \( Q \) by adding to \( P \) all the valuations that are needed in order not to favour any negative literal. The result of this operation I call the Positive Extension of \( P \), or \( \text{Pos}(P) \).

   For any \( P \), \( \text{Pos}(P) = \{ V \mid \text{there is a valuation } V' \text{ in } P \text{ such that } V' \subseteq V \} \) (recall that a valuation is seen as a set of atomic sentences)

   **Facts:** for any non negative proposition \( P \),
   1. If \( P \) is positive, \( \text{Pos}(P) = P \)
   2. \( \text{Exhaust}(P) \subseteq P \)
   3. \( \text{Exhaust}(P) = \text{Exhaust}(\text{Pos}(P)) \)
   4. \( \text{Pos}(\text{Exhaust}(P)) = \text{Pos}(P) \)

   From these facts, I prove the following theorem:

   **Theorem:** if \( P \) is a positive proposition, then \( \text{Max}(P) = \{\text{Exhaust}(P)\} \), and therefore \( P \) implicates \( \text{Exhaust}(P) \).

   **Proof:** let \( P \) be a positive proposition. Suppose \( i \in I(P) \), i.e. \( i \) is such that a speaker who is in information state \( i \) would choose \( P \) among the set of positive sentences. Then \( P \) must be the strongest positive proposition that \( i \) entails, e.g. \( P = \text{Pos}(i) \) and \( I(P) = \{i \mid \text{Pos}(i) = P\} \). Since \( \text{Pos}(\text{Exhaust}(P)) = \text{Pos}(P) = P \), \( \text{Exhaust}(P) \in I(P) \). Let \( i_1 \) be a member of \( I(P) \) such that \( \text{Exhaust}(P) \) does not entail \( i_1 \). Then there is a valuation \( V_1 \) in \( \text{Exhaust}(P) \) which does not belong to \( i_1 \). Therefore \( V_1 \) does not belong to \( \text{Exhaust}(i_1) \), and \( \text{Exhaust}(P) \neq \text{Exhaust}(i_1) \). But \( \text{Exhaust}(i_1) = \text{Exhaust}(\text{Pos}(i_1)) = \text{Exhaust}(P) \), which is contradictory. Hence there is no such \( i_1 \) and \( \text{Exhaust}(P) \) entails all the members of \( I(P) \), from which it follows that: \( \text{Max}(P) = \{\text{Exhaust}(P)\} \). Q.E.D

III. 4. Pair-list questions
Consider sentence (19) again (“Each of the students read *Othello* or *King Lear*”). If (19) is understood as an answer to a pair-list question like “Which students read which *plays by Shakespeare*?”, exhaustification predicts an exclusive reading for *or*. Note that the translation of a certain natural language sentence into a sentence of propositional logic will yield different results for different underlying questions. In the case of the above pair-list question, but not in other cases, atomic sentences represent elementary answers of the type ‘x read y’, and (19) will be translated as something like (19’):

\[(3') (A \lor B) \land (C \lor D) \land (E \lor F) \land \ldots \land (G \lor H)\]

Exhaustification of (19’) yields the desired result (exclusive reading for all the disjunctions). This context-dependency explains why judgments are not uniform.

**IV) Non positive propositions**

1) Negative answers do not lead to full exhaustification

Negative propositions can (trivially) be translated in a propositional language L’ so that their counterparts in L’ are positive: convert all negative literals of L into atomic sentences of L’. Thus if a negative proposition P is to be compared to all negative propositions, we predict exhaustivity for P with respect to its translation in L’, and, finally, we get the same result as Van Rooy’s second exhaustivity operator (the one that is used in decreasing contexts)

(36) a. – Among the professors and the students, who came?
   b. Less than two professors came

As a matter of fact, this is wrong, as (36)b. implicates indeed that one professor came, but nothing regarding non-professors.

It turns out, however, that we predict exactly the right results by defining alternatives of negative propositions as follows:

Let P be a negative proposition, then S(P) is the **closure under union and intersection of all the literals favoured by P**.

We predict "indirect implicatures" in Chierchia's sense:

(36’a). It is not the case that Mary and Peter will both come to the party
   b. It is the case that Mary or Peter will come to the party (but not both)

\[\neg (A \land B) \quad \neg A \lor \neg B\]

2. Answers that are neither positive nor negative

Let P be a proposition. We must distinguish two cases.

a) First case: some literals favoured by P are positive, others are negative, but there is no literal A such that both A and its negation are favoured by P
Example:
(37)a. Among philosophers, chemists and linguists, who will come?
   b. Three philosophers but less than two chemists will come.
   Implicates: “exactly one chemist and exactly three philosophers will come”

We can predict this result by assuming, again, that the alternatives of such a proposition P are the members of the closure under union and disjunction of all the literals favoured by P.

What do we predict for (38)?
(38) a. (Among Peter, Jack, Mary and Kim), who will come?
   b. Peter will come, but Mary won't come
   There should be no exhaustivity effect, which seems correct

b) Second case: some atomic sentences A are such that P favours both A and ¬A. What about (39)?

(39) context: there are five linguists, two chemists and two philosophers
   a. Who will come among linguists, chemists and philosophers?
   b. All the linguists except one
   c.((A∧B∧C∧D)∨(A∧B∧C∧E)∨(A∧B∧D∧E)∨(A∧C∧D∧E)∨(A∧C∧D∧E))∧(¬A∨¬B∨¬C∨¬D∨¬E)
   Literals favoured by (39)c. : A, B, C, D, E and all their negations. So if we keep the same procedure, we predict that c. should be compared to the boolean closure of \{A, B, C, D, E\}, and we should predict no implicature at all. b., however, seems to be read as exhaustive, e.g. as suggesting that no chemist and no philosopher will come.
   We can predict this result by stipulating that c's alternatives are the union of positive propositions and c itself.

(40) Exactly two students studied Maths or Linguistics

If (40)'s alternatives are defined as positive propositions plus (40), then (40) is read as exhaustive, e.g. implicates (41):
(41) Exactly two students studied Maths or Linguistics but not both, no student studied both, and no non-student studied either Maths or Physics

(42) All the linguists but one and two chemists read this book

If exhaustified, implicates:
All the linguists but one and exactly two chemists read this book, and nobody else

(43) a. Exactly two linguists and no chemist read this book
   b. Exactly two linguists but no chemist read this book
   c. Two linguists but no chemist read this books
   >does not implicate anything regarding philosophers.
It therefore seems that we need more refined tools in order to know how the alternatives are defined.

**Def 1:** A proposition $P$ **strongly favours** a literal $L$ if $P$ favours $L$ and $P$ does not favour the negation of $L$

**Def 2:** A proposition $P$ is **quasi-positive** if $P$ does not strongly favour any negative literal.

If we want to predict that only quasi-positive sentences lead to full exhaustification, we may adopt the two following rules, which cover all the cases:

**If $P$ is quasi-positive, $P$'s alternative set consists in the union of the set of positive propositions and $\{P\}$ itself.**

**If $P$ is not quasi-positive, then $P$'s alternative set consists in the closure under union and intersection of all the literals that $P$ favours.**

These rules make the following predictions:

(44) Between two and five linguists and no philosopher came.

>> No exhaustivity effect: nothing should be implicated regarding chemists

(45) Between two and five linguists and three philosophers came

>> Exhaustivity effect: suggests that no chemist came

(46) Three philosophers but less than two chemists came

>> No-exhaustivity effect: nothing should be inferred regarding linguists.

Though judgments are not so clear, an informal inquiry seems to indicate that most people have the expected intuitions. More work needs to be done in order to understand what is really going on here.

**V) Towards an accounts in terms of a few basic cognitive principles?**

(38) a. Who will come among Peter, Jack, Mary and Kim?
   b. Peter will come, but Mary won't come

**Epistemic principle of symmetry:**
If the speaker’s information state $i$ is such that $i$ favours two negative literals $L$ and $L'$, then the proposition $P$ expressed by his utterance must treat $L$ and $L'$ identically, i.e. must favour both or none of them

**Preference for positive information**
If the speaker’s information state $i$ favours a positive literal $L$, then the proposition $P$ expressed by his utterance must favour either $L$ or its negation:
Case where the negation of a favoured positive literal is favoured:

(47) Less than two chemists came

(47) favours all the negative literals corresponding to “$x$ didn’t come”, where $x$ is a chemist: But it implicates “one chemist came”. In fact, the speaker’s information state
in that case favours both positive and negative literals of the form “(not) x came”, with x a chemist.

**Conclusion**

To be worked out:

- *some* as a **vague** cardinal
- conditional perfection: inference from “If it rains tomorrow, Mary will go the the library”, to “Mary will go to the library if and only if it rains tomorrow”, from which the arial of indirect implicatures is straightforwardly predicted. As Van Rooy noticed, this strengthening can be seen as a case of exhaustive reading once we consider *if* as a quantifier over worlds.

If this view is correct, we predict a correlation between *if-and-only-if* readings and indirect implicatures in conditional sentences.

- the effects of *focus*

- A sentence like "John will come or John and Mary will" expresses the same proposition as "John will come", but is not interpreted in the same way. This shows the limitations of any procedure that only takes into account the literal semantic values of sentences, and not their actual phonological and syntactic form.

- It remains to be seen whether the role played by polarity (namely, the distinction between positive and non-positive answers) can be derived in a more principled way. I suspect that introducing a notion of *utility* in our model of information processing, as Nilsenova & Van Rooy (2002) do in order to account for the pragmatic effects of polar questions, could help explain why "positive" and "negative" answers pattern asymmetrically

- The procedure I have defined is context-dependent (since implicature computation depends on what the question under discussion is), but it is possible to devise a very similar procedure that would not be context-dependent. These two procedures, taken together, can provide us with an analytic tool for investigating to what extent scalar implicatures are *generalized* rather than extremely sensitive to context.

- Chierchia has not been proved wrong. Many arguments for locality come from the interaction of scalar implicatures with other aspects of grammar: intervention effects with NPIs, presupposition projection. I have addressed none of them.

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