Compositionality and Molecularism

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This paper is devoted to the analysis of the conceptual interplay between the principle of compositionality (PC) and the claim that meaning is molecular rather than holistic. I first argue that some sort of molecularity thesis being true is a necessary condition for arguments in favor of the principle of compositionality to make sense. Then I propose a formal characterization of molecularity which aims at taking into account the way it interacts with compositionality, as captured by Hodges setting, to (help) explain our mastery of language.

1 Introduction

Hodges, in his seminal 2001 paper *Formal Features of Compositionality*, provides a setting for a precise definition of compositionality, in which various questions about the possibility of finding compositional semantics for languages are successfully addressed. This fact suggests that this setting could be used for the formalization of further strengthening of compositionality, for which, pace (Hodges 1998), compositionality is indeed the problem. Various approaches have alreadt been suggested. Pagin (2003) advocates some kind of reverse compositionality, Kracht (1998) puts forward a notion of strict compositionality with built-in computability requirements. Our approach is based on a notion of compositionality which encapsulates anti-holist requirements. The first section is devoted to conceptual arguments in favor of this one strengthening. In the second section, Hodges’ setting will come into the picture and prepare the ground for the formalization of our strong notion of compositionality in the third section, based on an extension of Hodges’ mathematics.

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2 Compositionality and Molecularity

A proposal for strengthening (PC)

Among the many conceptual and technical issues surrounding compositionality, two of them are especially noteworthy and enduring:

- methodological status. What is the status of the principle of compositionality for formal or natural languages: is it substantial or empty? is it a necessary condition for a language to be understandable? Must any semantics be compositional?

- formalization. One seeks a definition of compositionality in a given mathematical framework, and then tries to investigate its properties, such as conditions for existence, possible strengthening and so on (see e.g. Janssen 1997).

It has been a temptation to rest on formal results about compositionality in order to sustain various theses concerning the status of the principle. In particular, the temptation has been to rely on results establishing that a compositional semantics is always available – the most famous such result being Zadrozny’s (1994) – and to argue for the non-substantiality of the principle. These attempts have been widely criticized on the ground that not any compositional semantics was interesting\(^1\). So the upshot of formalization has been a little bit disappointing, and even Hodges’ conclusion for example was that, after all, compositionality was not the problem.

The lesson to be drawn is first that one should start the other way around, and ask why compositionality is interesting, on which theoretical grounds it is needed, before going back to formalization, and second that one should look for strengthening of the notion of compositionality that would protect availability results from triviality.

There are two main arguments in favor of PC:

- the argument from productivity. Competent speakers can understand a complex expression \(e\) they never encountered before. So there must be some knowledge which is responsible for this ability. As all they have to start from to grasp the meaning of \(e\) is the syntactic structure of \(e\) plus its simple constituents, they must use their knowledge of the meaning of these constituents and of the associated syntactic links of combination.

- the argument from systematicity. Anyone who understands \(e\) understands also, without any additional information \(e'\) if \(e'\) is built out of the same

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\(^1\)WESTERSTÅHL (1998) convincingly shows the limits of Zadrozny’s claims, but Zadrozny himself recognizes that his result shows only the need of adding new constraints on what kind of compositionality is interesting.
constituents as $e$, and $e$ and $e'$ are both meaningful. The only plausible way this is possible is if the meaning of $e$ was known thanks to the knowledge of the meaning of its constituents and syntactic links, so that the meaning of $e'$ could be computed in an analogous way.

Both arguments have a common structure: compositionality is taken to be the best explanation of facts about speakers competence. But I shall argue that both arguments share a common presupposition: compositionality is the best explanation only if some kind of molecularism is true, that is, only if the meanings of basic lexical items and syntactic constructions can be determined on a finite and local basis. This is because compositionality plays the role of best explanation only if it performs some sort of epistemic reduction, from knowledge of sentence meaning to knowledge of word meaning.

In order to establish this point precisely, let’s consider the following theses:

**Principle of Compositionality (PC).** The meaning of a complex expression is a function of the meaning of its parts and of the syntactic rules by which they are combined.

**Epistemic version of the Principle of Compositionality (EPC).** Knowledge of the meaning of a complex expression is attained through knowledge of the meaning of its parts and of the syntactic rules by which they are combined.

**Epistemic version of Frege’s context principle (EFCP).** Knowledge of the meaning of basic expressions is attained through knowledge of the use of sentences containing them.\(^2\)

**Molecularism (M).** As long as the meaning of basic expressions gets determined by their use in sentences, it is sufficient to consider the use of certain sentences – the meaning fixing ones – so that it is possible to learn the language step by step.

**Holism (H).** As long as the meaning of basic expressions is determined by their use in sentences, it is necessary to consider the whole use of sentences in the language, so that it is not possible to learn only one part of the language.

We sustain the following claims:

1. (EPC) implies (PC), but the converse is not true.

2. (EPC) – and not (PC) – explains productivity and systematicity.

\(^2\) (EFCP) is an epistemic version of Frege’s context principle, if it construed as (FCP): The meaning of basic expressions gets determined by their use in sentences.
3. Modulo (EFCP), (H) is not compatible with (EPC), though it is compatible with (PC).

4. Modulo (EFCP), (PC) + (M) strongly supports (EPC).

Point 1. is clear: (EPC) states that some dependence exists, which is epistemically exploited, so that it implies (PC). But, in the other direction, the functions involved in (PC) could be non computable so that the shift from (PC) to (EPC) is not valid.

As to 2., it should be clear from the arguments for productivity and systematicity that what they do point at is (EPC) and not (PC). In both cases, one needs to account for the understanding of some complex expressions, but this knowledge can be explained only if the dependence between meanings has some epistemic bearing, as in (EPC). (PC) in itself is not interesting when the concern is with actual linguistic competence; it will be interesting only as long as it is a basis for an epistemic reduction, in the sense that it reduces the problem of knowing the meaning of a new complex expression to a simpler problem about knowledge of the meanings of basic terms. This epistemic reduction is phrased by (EPC).

Because of 1. and 2., it is already clear why (PC) could be misconstrued as the best explanation of productivity and systematicity; but 2. and 3. shows that it is only part of the story, that is, (PC) is not sufficient to explain productivity and systematicity.

The argument in favor of 3. rests on a charge of circularity. By (EPC), knowledge of the meaning of a certain complex sentence S depends on knowledge of the meaning of its basic parts. But then, by (EFCP), knowledge of the meaning of these parts depends on knowledge of the use of sentences containing them. Finally, by (H), this knowledge involves knowledge of the use of the very sentence S itself, so that grasping the meaning of S is shown to presuppose grasp of the very meaning of S. This is not to be confused with a contradiction between (PC) and (H): one can perfectly well imagine a compositional language such that change in the meaning of any simple or complex expression induces changes on the meaning of every other expression.

We have seen that there is a epistemological gap between (EPC) and (PC), and more generally, there is no hope to find an absolutely conclusive path from formal properties of languages to (EPC), which, after all, is an empirical matter about the actual working of our mastery of language. But nevertheless, the whole point of discussions about (PC) is to find formal constraints on languages on behalf of certain features of their use, like productivity and systematicity. 3. suggests the direction in which (PC) should be strengthened: 4. states what this direction is. The intuition for 4. is that the charge of circularity which has appeared in the discussion of 3. is leveled if (H) is false and (M) is true.
In that case, it is possible to have a clear picture of how language learning can proceed: even if mastery of sentence meaning is prior to mastery of word meaning, (PC) might not be deprived of explanatory power, because the basic mastery of sentence meaning with which we cannot dispense is easier to get than the whole mastery of language to which the mastery of word meaning gives access.

The epistemological gap is not bridged, in the sense that (PC) + (M) still do not imply (EPC), but (PC) + (M) are viable candidates for formal constraints on languages, because (PC) + (M) supports (EPC) and (EPC) seems the best explanation of essential facts concerning language use. In the particular case of the argument from systematicity, it presupposes that there is something special with the expression $e'$, namely that its meaning is interdependent with the meaning of $e$. But if meaning holism is true, this interdependence between $e$ and $e'$ is nothing special: such an interdependence holds between any two sentences of the language, so that there is nothing to explain, and compositionality is not needed either. This justifies Dummett’s claim:

The principle of compositionality is not the mere truism [...] that the meaning of a sentence is determined by its composition. Its bite comes from the thesis that the understanding of a word consists in the ability to understand characteristic members of a particular range of sentences containing that word. (Dummett, 1991, p. 225)

**The formalization problem**

A large part of the interest in formalizations of (PC) was lying in the fact that it could be used as a criterion of admissibility for possible theories of language. A theory failing the test of (PC) would be a theory of a language which would fail to explain basic facts of our mastery of this language which are closely connected to our ability of learning it. Under the hypothesis that these facts hold or that the language this theory is about is learnable, such a theory should be rejected. If the preceding argument is correct, these very facts support not only (PC), but a stronger property of languages, namely (PC) + (M). Our aim in the rest of this paper will thus be to get a formal grip on (PC)+(M) and provide a stronger test for theories of language.

How should this formalization proceed? First, note that there is some ambiguity in the phrasing of the principle of molecularity. On one hand, it can be seen as a static property of a semantics for a language: meanings for the whole language can be seen as inherited from meanings of small parts of the language, that meanings are preserved from parts to the whole. Slightly more formally, a language $L$ together with a semantics $\mu$ is taken to be molecular if it is possible to analyze it into proper sublanguages $L_i$, such that $\mu$ agrees with the semantics...
for the \( L_i \) on terms belonging to the \( L_i \). We will say that a language together with a semantics enjoys static molecularity if \((PC)+(M)\) holds for it, \((M)\) being taken in a static sense. On the other hand, molecularity is also a dynamic thesis pertaining to language learning: it is the claim that language learning proceeds incrementally, or step by step. The basis on which meanings are determined should be structured in such a way that it is possible to learn meanings of terms independently of each other, or at least in a cumulative way, such that new semantic knowledge does not imply revision of previous semantic knowledge. Dynamic molecularity is a property not of semantics but of linguistic materials on which language learning is based. This second sense of molecularity is stronger than the second one: a molecular picture of the materials of language learning for \( \langle L, \mu \rangle \) (dynamic molecularity) should yield an analysis of language \( \langle L, \mu \rangle \) (static molecularity).

Here comes a first problem. In its strong dynamic sense, \((M)\) lacks content until an answer is provided to the problem of how word meaning gets determined (determination problem). Paraphrasing Dummett, \((PC)\) loses its bite if no molecularist answer to the problem of the determination of word meaning is provided. But \((M)\) is not by itself such an answer; it is only a constraint on possible answers to the determination problem.

We shall choose a particular type of answer to the determination problem and side with proponents of inferential role semantics (IRS), according to whom what matters is the acceptance of certain sentences and inferences. More precisely, following Greenberg and Harman, (IRS) is a special case of conceptual role semantics. A conceptual role semantics is “any theory [of meaning] that holds that the content of […] symbols is determined by any part of their role or use in thought” (Greenberg & Harman, in press, p.1). (IRS) is then a special case of conceptual role semantics, according to which “the recognition of internal inferential and implicational relations [is taken] to be crucial to the meaning” (Greenberg & Harman, in press, p.2) of symbols.\(^3\) Other non inferen-
tialist options would be perfectly acceptable: Molecularism and inferentialism are two independent theses. One can stick to inferential role semantics while embracing holism. For example, this is the kind of position endorsed by Hartry Field (1977). Conversely, nothing in molecularism *per se* forces one to be an inferentialist.

Starting with a definition of static molecularity, we will look for a characterization of dynamic molecularity. More precisely, the aim will be to capture the features of inferential roles which guarantee static molecularity; therefore the possibility to derive static molecularity from dynamic molecularity will be the touchstone of our tentative characterization of dynamic molecularity.

Here comes a second problem. Inferential roles are given through semantic rules, giving rise to some consequence relation. The basic material of an IRS consists thus in a relation between sentences. As a consequence, IRS directly supports a semantics only for the sentential part of a given language. In contradistinction to that, compositionality and static molecularity are a property for full semantics, because static molecularity is concerned with the semantics for every kind of terms of the language. Here comes into play the results of Hodges (2001) which aim precisely at bridging the gap between word meaning and sentence meaning.

The full picture should thus include a characterization of a property of inferential roles for a language, such that they constrain sentence meanings in a way that guarantees the holding of static molecularity. This picture is intended to support (EPC), because it should make the following plausible: Understanding of words comes from understanding of sentences, as constrained by acceptance of *certain* inferences; and it is on the basis of the understanding of word meaning that the full range of sentences can be understood. In a nutshell, the underlying claim behind the formalization to come is that the core idea of compositionality as the explanation of productivity and systematicity is the interplay between sentence meaning and word meaning.

Let’s finally discuss a possible objection to our presentation of the formalization problem. If no constraint is placed upon the way the semantics is to be analyzed in the definition of static compositionality, enjoying static compositionality is bound to be trivial. As a consequence, our distinction of two versions of molecularity might seem at best unnecessary and at worse misleading. In contrast, Pagin (1997) states clearly that holism refers to a property of theories of meaning. It involves both semantic facts and facts pertaining to meaning determination: certain non-semantic properties of linguistic expressions are pinpointed, and these properties determine the meaning of the expressions. A theory of meaning will be holistic if, and only if, the whole extension of the

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in favor of molecularism does not commit us to the adoption of his particular version of IRS.
non-semantic property is relied upon and the meanings get determined together, in the sense that assignments of meanings to different expressions are interdependent. But in fact this is perfectly consistent with our setting: static molecularity is interesting because it describes a semantic property which makes (EPC) possible, but the analysis of language underlying molecularity must come from some actual determination of meaning, i.e. static molecularity must hold as the result of the holding of dynamic molecularity, if one accepts as relevant non-semantic facts the inferential roles.

3 Hodges Setting

The setting of Hodges (2001) is well-suited to formalize the strong version of compositionality we are interested in, even if it is not primarily designed to do that. The results concerning the extension problem – i.e. how to get from a (sort of) compositional semantics for a small language or for the sentential part of a language to a compositional semantics for a bigger language or for the whole language – paves the way for a general definition of static molecularity and for a result linking dynamic and static molecularity. To begin with, we will recall these results.

**Preliminary definitions**

*Language and term algebra*

A language L is a triple \(\langle E, A, \Sigma \rangle\) where

- \(E\) is the set of expressions
- \(A \subseteq E\) the set of atomic expressions
- \(\Sigma\) a set of syntactic rules, that is partial maps from \(E^n\) to \(E\).

To each \(L\), one can associate a grammatical term algebra \(GT(L)\), which is a subset of the term algebra over \(L\). From an intuitive point of view, it corresponds to the structural analysis of the expressions in \(L\) which disambiguates them.

*Semantics and synonymy relations*

A semantics for \(L\) is a map \(\mu\) whose domain is a subset of \(GT(L)\). A synonymy for \(L\) is an equivalence relation \(\equiv\) on a subset of \(GT(L)\). One associates to every semantics \(\mu\) the synonymy \(\equiv_\mu\) induced by in the usual way; conversely, a synonymy relation \(\equiv\) gives rise to a semantics \(\mu_\equiv\) which interprets each term by its synonymy class. Two semantics are equivalent if they have the same associated synonymy.
We say that $\mu$ is as fine-grained as $\mu'$, notation $\mu \leq \mu'$ iff $\exists' \mu \subseteq \equiv' \mu'$ where $\equiv'$ is obtained from $\equiv$ by restricting it to the intersection of the domains of $\mu$ and $\mu'$.

We say that $\mu$ preserves $\mu'$ iff the domain $D$ of $\mu'$ is a subset of the domain of $\mu$ and $\mu \leq \mu'$ and $\mu' \leq \mu$. $\mu$ preserves $\mu'$ iff $\mu$ is equivalent to an extension of $\mu'$ in the usual sense.

Two terms $p$ and $q$ are separated by a context $t(x)$ in $\mu$ iff $t(p/x) \neq \mu t(q/x)$. Two terms are separated in $\mu$ iff there is a context which separates them.

Two terms $p$ and $q$ have the same $\mu$-category (notation $p \sim \mu q$) iff replacing one by another preserves meaningfulness with respect to $\mu$.

A semantics $\mu$ is $\mu'$ husserlian iff for all terms $p,q$, if $p \equiv \mu$ then $p \sim \mu q$ (that is, $\mu$-synonyms have the same $\mu'$-category). Compositionality

A semantics $\mu$ is compositional if there exists a function $r$ such that for every complex $\mu$-meaningful term $s = \alpha(t_1...t_n)$, $\mu(s) = r(\alpha, \mu(t_0)...\mu(t_n))$. Intuitively, the functions $r_\alpha$ tell how the meaning of a complex expression $\alpha(t_1...t_n)$ depends on the meaning of its constituents $t_1...t_n$ plus the syntactic way they are combined, $\alpha$.

It is useful to isolate a weaker property, which for a husserlian semantics, is equivalent to compositionality, 1-compositionality: $\mu$ is 1-compositional iff $p \equiv \mu q$ implies $s(p/x) \equiv \mu s(q/x)$.

**The extension theorems**

The main idea introduced by Hodges (2001) is the notion of Fregean extension, which captures what should be the semantics $\nu$ for the expressions of a language, given a semantics $\mu$ for the sentences of a language. It is intended to capture the Fregean idea, expressed in Frege’s context principle, that the meaning of a term of the whole language is the contribution that the term makes to the meaning of the sentences containing it.

The four conditions are:

a) $\nu$ is $\mu$-husserlian

b) $\mu$ is 1-compositional with respect to $\nu$ synonyms.

c) If two terms are not $\nu$ synonyms, there must be a context which separates them in $\mu$ (full abstraction over $\mu$).

d) $\nu$ is an extension of $\mu$.

Hodges’ results are existence results:

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4The theorems do not involve $GT_L$ and $S_L$ but $GT_L$ and any subset $CGT_L$ cofinal to $GT_L$. $CGT_L$ is cofinal to $GT_L$ if, though $CGT_L$ is smaller than $GT_L$, every expression in
Theorem 1 (First Extension Theorem, Hodges 2001). Let $L$ be a language, $GT(L)$ the set of grammatical expressions of this language, $S_L$ the subset of $GT(L)$ corresponding to the set of $L$ sentences, $\mu$ a husserlian and 1-compositional semantics for $S_L$, there exists a semantics $\nu$ for $GT(L)$ which is a total Fregean extension of $\mu$ and which is compositional and unique up to equivalence.

Hodges first extension theorem explains how we can get from sentence meaning to word meaning, in so far as it explains how a satisfactory compositional semantics for the whole class of grammatical terms can be devised on the basis of a well-behaved semantics for sentences.\(^5\)

Theorem 2 (Second Extension Theorem Hodges 2001). Let $L$ and $L'$ be two languages, $GT(L)$ and $GT(L')$ the set of grammatical expressions of these languages, $S_L$ and $S_{L'}$ the subsets of $GT(L)$ and $GT(L')$ corresponding to the sets of sentences of these languages, $\mu$ and $\mu'$ two husserlian and 1-compositional semantics for $S_L$ and $S_{L'}$, let $\nu$ and $\nu'$ the two semantics for $GT(L)$ and $GT(L')$ given by Theorem 1,

- If $\mu \leq \mu'$ (and $\sim \mu \upharpoonright S_L \subseteq \sim \mu'$), then $\nu \leq \nu'$
- If any two terms $p$ and $q$ of the same $\mu$ category which are separated in $\mu$ are already separated in $\mu'$ – we shall say that the separability property holds – (and $\sim \mu' \subseteq \sim \mu$), then $\nu' \leq \nu$

The idea behind the second extension theorem is that since the meanings of the whole set of terms are determined by the meanings of sentences – as set up by the definition of Fregean extension – the degree to which sentence meanings are kept fixed as the language gets bigger determines the degree to which term meanings are kept fixed. Therefore, Hodges theorem can be seen as capturing, at least partially, the idea of a step by step growth of the language, because it can represent the semantics for the whole language as (quasi-)extensions obtained from semantics for parts of the language.

Since this second theorem is the most interesting to us, we shall spell out the proof in details, so that the way this determination works is clear:

\(\text{GT}_L\) is a subexpression of an expression in \(\text{CGT}_L\). Cofinality to $\text{GT}_L$ is usually taken to be a necessary condition for a subset of $\text{GT}_L$ to be the set of sentences of $L$, for any $L$, though a necessary and sufficient syntactic characterization of $S_L$ is still lacking. Since the only subset of $\text{GT}_L$ we will be interested in, we have rephrased Hodges’ theorems and dropped talk of cofinality.

\(^5\)Actually, this too involves some drastic idealization, because even though word meaning might be acquired only in the context of their use of these words in sentences, other mechanisms – such as innate sensibility to various kind of objects – obviously come into play at this very stage.
Let \( p \) and \( q \) be in \( GT(L') \), assume \( p \equiv \nu q \), we want to show that \( p \equiv \nu' q \). From \( p \equiv \nu q \), we get

1. \( p \sim \mu q \) by condition a) of the definition of Fregean extension

2. for all contexts \( s(x) \) yielding \( \mu \)-meaningful sentences for \( p \) and \( q \),
   \[ s(p/x) \equiv \mu s(q/x) \] by condition b) of the definition of Fregean extension

From that we get

1'. \( p \sim \mu' q \) by (1) and \( \sim \mu \uparrow S_L \subseteq \sim \mu' \).

2'. for all contexts \( s(x) \) yielding \( \mu' \)-meaningful sentences for \( p \) and \( q \),
   \[ s(p/x) \equiv \mu' s(q/x) \] by (2) and \( \mu \leq \mu' \)

By condition c) of the definition of Fregean extension, this implies that \( p \equiv \nu' q \).

Similarly, let \( p \) and \( q \) be in \( GT(L') \), assume \( p \equiv \nu' q \), we want to show that \( p \equiv \nu q \). From \( p \equiv \nu' q \), we get

1. \( p \sim \mu' q \) by condition a) of the definition of Fregean extension

2. for all contexts \( s(x) \) yielding \( \mu' \)-meaningful sentences for \( p \) and \( q \),
   \[ s(p/x) \equiv \mu' s(q/x) \] by condition b) of the definition of Fregean extension

From that we get

1'. \( p \sim \mu q \) by (1) and \( \sim \mu' \subseteq \sim \mu \).

2'. for all contexts \( s(x) \) yielding \( \mu \)-meaningful sentences for \( p \) and \( q \),
   \[ s(p/x) \equiv \mu s(q/x) \] by (2) and the separability property.

By condition c) of the definition of Fregean extension, this implies that \( p \equiv \nu q \). We note here that this second proof is not exactly the dual of the first one: to get from (2) to (2'), \( \mu' \leq \mu \) is not sufficient, because there are new contexts in \( S(L) \) for which \( \mu' \leq \mu \) says nothing, and that’s why the separability property, which is stronger than \( \mu' \leq \mu' \), is needed.

\(^6\)The separability property implies \( \mu' \leq \mu \). Assume that \( p \equiv \mu q \) and \( p \not\equiv \mu q \). \( x \) is then a separating context in \( \mu \) for these two terms, by the separability property, there is a context \( s(x) \) such that \( s(p/x) \not\equiv \mu s(q/x) \), but, since \( \mu' \) is assumed to be 1-compositional, this contradicts \( p \equiv \mu' q \). It is easy to see that \( \mu' \leq \mu \) is compatible with the separability property not holding.
4 Formalizing Molecularity and Compositionality Together

Even if the notion of Fregean extension explains how sentence meaning determines word meaning, this does not imply that it provides an answer to the determination problem, that is the problem to determine on which learnable basis meanings get known. In order to be able to apply Hodges extension theorems, one has to start with meanings for an infinite set of sentences, therefore an infinitary access to meaning is still presupposed. To answer the determination problem, one has explain how we can get to know, on a finite basis, sentence meaning. Here comes into play inferential role semantics, which explains how meaning is already constrained at the level of sentences.

So we still have to formalize static molecularity in Hodges setting – to be done in the first subsection – to integrate inferential roles into Hodges’ account of meaning – second subsection – and – this is the crucial point – to put these two things together in order to show what a strong account of compositionality based on molecularism and inferentialism consists in. This will be done in the third subsection, through a definition of dynamic molecularity as the counterpart in terms of inferential roles of static molecularity.

Formalizing compositionality plus static molecularity

Molecularity is the thesis that language can be learned step by step. Considering things the other way around and from a static point of view, this implies that a language can be cut into simpler parts such that meaning is roughly preserved from the simpler parts to the whole language. Given a language $L$ and its set of grammatical expressions $GTL$, one can imagine two kinds of analysis of $GTL$ into simpler languages:

- We can drop a few basic expressions of $AL$ (the set of atomic expressions of $L$) in order to see it as the addition of a few new expressions determined in a certain way to an already determined language. This is a shift from $GTL$ to $GTL'$ where $AL \supseteq A_L'$ and $AL \setminus A_L'$ is required to be as small as possible.

- We can see $GTL'$ as the union of already determined smaller languages. This is a shift from $GTL$ to $\{GTL_i\}_{i \in I}$, where $AL = \bigcup\{AL_i\}_{i \in I}$.

Intuitively, the first case corresponds to an enrichment of the language, new expressions being added together with new semantic rules as implicit definitions. For example, $GTL'$ could be a purely logical language and $GTL$ the language of arithmetic. The second case amounts to dividing up the language into separate parts. For example, one of the $GTL_i$ could be the mathematical part of language and another one of them could be the purely physical talk consisting in the ascription of everyday life properties to macroscopic objects.
The two cases should be distinguished, because in the first one, some new information on meanings must be available, so that the speaker can understand the meaning of expressions with occurrences of basic expressions among $A_L/A_{L'}$. In the second case, no new information should be required to grasp the meaning of sentences of the whole language. Intuitively, it should be possible to consider $\nu$ as completely determined by the $\nu_i$. If special conditions hold, namely if $\nu$ extends the $\nu_i$, is compositional and if synonyms can be found at will across the $GT(L_i)$, the meaning of every new term of $GT(L)$ is indeed already determined.\(^7\)

From this top-down perspective, molecularity for a language $L$ amounts first to the possibility of performing such analysis on $L$ until one reaches the empty language:

**Definition 1 (Molecular Compositional Tree).** A molecular tree for a language $L$ is a tree constructed from the root $GT_L$ by applying one of the two operations of analysis just defined, such that at each node, the sublanguage is compositional and the end nodes are the empty set.

But of course, molecularity encapsulates at the same time the thesis that through such an analysis, meanings are roughly preserved. Here the point is to agree on which kind of rough preservation of meaning we require.

Given a molecular tree for a language $L$ labeled with semantics $\nu_s$ for the sublanguages at each node $s$, let’s say that the property (a) holds at $s$ iff for every node $sa$ immediately below $s$, $\nu_s \leq \nu_{sa}$, and that the property (b) holds iff $\nu_s \geq \nu_{sa}$.

The following definitions are intended to capture the various kind of requirements that one may impose upon a molecular tree to be a molecular analysis.

**Definition 2 (Static Molecular analysis).** A molecular compositional tree for a language $L$ with a semantics $\nu$ for $GT_L$, labeled with semantics $\nu_s$ for the sublanguages at each node $s$, is

\[^7\]Assume that for every $\alpha(b_1...b_n) \in GT(L)$, there exists $i \in I$ and $b'_1...b'_n$ such that $\alpha(b'_1...b'_n) \in GT(L_i)$ and $b_k \equiv \nu b'_k$. Then, we do have $\nu'((\alpha(b_1...b_n)) = \nu_i((\alpha(b'_1...b'_n)))$, since by compositionality of $\nu'$, $\nu'((\alpha(b_1...b_n)) = \nu'((\alpha(b'_1...b'_n)))$ and $\nu'$ is an extension of $\nu_i$. This means that the functions $r_\alpha$ interpreting the syntactic rules $\alpha$ of $L$ are determined by those doing the same job for the $L_i$. More generally, what most probably happens in real life cases, is that the values of the $r_\alpha$ for unknown inputs is inferred by some kind of induction. For example, if you learn to interpret compositionally conjunction as intersection on a small language, you go on interpreting it the same way when you have new semantics values as inputs. Unfortunately, the formalization of this point, which would involve an account of rule-following, goes beyond our present aims and possibilities.
• a strong static molecular analysis of $L$ iff at every node $s$, properties (a) and (b) hold.
• a cumulative static molecular analysis of $L$ iff at every node $s$ property (a) holds.
• an anti-cumulative static molecular analysis of $L$ iff at every node $s$ property (b) holds.
• a weak static molecular analysis of $L$ iff at every node $s$ either property (a) or property (b) holds.

Of course, we take a language to enjoy static molecularity if and only if it has a molecular analysis. But the last definition shows that static molecularity is not an all or nothing concept: one can imagine many ways in which meanings are preserved from parts to the whole. The question is: which notion of molecular analysis do we need in order to support (EPC)?

Precisely on the ground of (EPC), Dummett’s position is in favor of a strong interpretation of molecularity. But it has been argued that such a strong conclusion is not justified (Pagin 1997; Pagin 200?). Let’s restate Pagin’s argument in our terms. First, only a very strong version of holism implying the thesis Pagin has labeled the Total Change Thesis according to which any change of acceptance of sentences or inferences will change the meanings of the sentences, threatens (EPC). This means that it is perfectly possible to accept a weaker form of holism without being committed to the charge of circularity developed in the first section. It is clear that if the Total Change Thesis is true, understanding any sentence requires understanding every sentence, so that the detour provided by word meaning is epistemically useless. But, following Pagin, it is perfectly compatible with holism to refuse the Total Change Thesis so that:

the meanings assigned to my expressions at time $t_i$, based on the set $\Sigma_i$ of sentences and inferences that I accept then, are the same as the meanings assigned to my expressions at time $t_k$, based on the new set $\Sigma_k$ of sentences and inferences I accept at $t_k$, provided my decisions between $t_i$ and $t_k$ are normal. (Pagin, 1997, p.30)

If this is correct, grasp of the meaning of basic terms in the context of a sublanguage would guarantee reliable knowledge of their contribution to meanings of expressions in wider contexts of use.

Another solution is to reject the black and white picture Dummett wants us to draw. Though perfect knowledge of meaning may depend on total acquaintance with the whole set of accepted sentence and inferences, partial acquaintance with this set might well yield partial knowledge of meaning, so that understanding of basic expressions in the setting of a fragment of the whole language...
would not be irrelevant to understanding of sentences containing them in the whole language. This view has been advocated by Bilgrami (1986) and more recently by Dresner (2002); these authors interpret it as a rejection of Dummett’s requirement of molecularity, we prefer to give it a dual interpretation according to which what is sought for is a weaker notion of molecularity.

The common strategy behind these counter-arguments against Dummett is to enhance the role played by regularities among meanings even in a holistic setting. The point is that even if (EPC) is not be literally true, some weaker version of it might be sufficient to explain the acquisition of our semantic competence. Even though fully grasping the meaning of a complex expression might involve irreducible knowledge about its use, knowledge of the meanings of basic terms of a complex expression, as determined by their use in other contexts, could do a significant part of the job and yield rough knowledge of the meaning of the complex expressions, where rough would mean either reliable though defeasible or partial and incomplete.

How does this fit with the various kind of molecular analysis we defined? Dummett’s position is captured by the notion of strong molecular analysis, because in this kind meanings are kept fixed along the tree. Without further ado, the partiality view seems to correspond to the notion of weak molecular analysis: the synonymy relation gets refined or new synonymy are discovered, but it assumed that this process allows for some kind of progressive grasp of meaning; acceptance of the cumulative or anti-cumulative view would stem out of the claim that only one of this two processes takes place, because of some special features of language learning. The reliabilist view is captured also by the weak notion, or perhaps more adequately by some mixed of the strong and the weak notions: the strong case would be the rule, although some local exceptions are allowed to take place, according to the weak notion. In the rest of this paper, our aim will not be to adjudicate between these competing stories about molecularity and the possibility of language, but to see how they fit in the compositionality view about meaning and to see which constraints on an Inferential Role Semantics ensue from them.

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8Dresner (2002) thinks that the partiality view supports the shift from a model-theoretic account of meaning to an algebraic one. Unfortunately, Dresner is very sketchy as to how this algebraic setting can be put to good use. The approach we develop here is clearly related to Dresner’s, as long as it leaves room for the partiality view, the main difference being that we are not committed to a determinate way of representing meanings, since we work in Hodges setting in which, in a very Quinean spirit everything is dealt with thanks to the synonymy relation, bypassing the representation problem.
Adding inferential roles

Hodges (2001) provides an account of how sentence meaning determines word meaning. We have just sketched an account of how molecularity works for word meaning. To push further and characterize dynamic molecularity, we still need an account of how sentence meaning is constrained by meaning-constitutive inferences.

So now we add a representation of inferential roles. These roles consist in acceptance of certain sentences or inferences. As a consequence, they directly determine meaning only at the sentence level: we have then to explain how this direct determination works and Hodges result will provide for free the understanding of how it bears more generally on term meaning.

The intuition is that semantics for a given language are constrained by the acceptance of certain inferential roles for the terms of this language: any semantics for a language must respect the meaning relations underlying the recognition of these roles. Let’s then define a constrained language $S_L = \langle S_L, \vdash_L \rangle$, as a set of sentences of a language $L$ equipped with a consequence relation on $\wp(S_L) \times S_L$ induced by the set of semantic rules $R_L$ of $L$. The notion of semantics is enriched in order to take into account the consequence relation: it is an ordered set $\langle M, \preceq \rangle$ and a function $\mu : S_L \rightarrow M$ such that if $\phi \vdash \psi$, then $\mu(\phi) \preceq \mu(\psi)$. $\preceq$ is intended to mirror at the semantic level the syntactic consequence relation: meanings are equipped with an entailment relation.

First, let’s make clear some points concerning this enriched notion of semantics and ask the two following questions: given a constrained language $S_L$ 1) what are sufficient conditions for the existence of a 1-compositional semantics for $S_L$? and 2) is there something like the good semantics for $S_L$?

We shall then prepare the ground for the definition of dynamic molecularity by asking: given a constrained sublanguage $S_L'$ of $S_L$ with semantics $\mu'$, 3) what are the conditions for the existence of a (unique) compositional semantics $\mu$ for $S_L$ such that $\mu \preceq \mu'$? and conversely 4) what are the conditions for the existence of a (unique) compositional semantics $\mu$ for $S_L$ such that the separability property holds between $\mu$ and $\mu'$?

Let’s start with a few more or less standard definitions.\footnote{Definition 3 matches Tarski’s well known definition of a consequence relation. Definition 4 is intended to yield 1-compositionality, and it is generalized in definition 7. Definitions 5 and 6 are fairly standard.}

**Definition 3 (Well-Behaved Language).** A constrained language $S_L$ is well-behaved iff $\vdash_L$ is reflexive and transitive.\footnote{Strictly speaking, we mean that the relation on $S_L \times S_L$ induced in the expected way by $\vdash_L$ is reflexive and transitive.}
Definition 4 (Equality Friendly Language). A constrained language $S_L$ is equality friendly iff $\phi \vdash_{L} \psi$ and $\psi \vdash_{L} \phi$, then for all meaningful contexts $s(x)$ for $\phi$ and $\psi$, $s(\phi/x) \vdash_{LS}(\psi/x)$.

Definition 5 (Conservativity). Given two constrained languages $S_L$ and $S_{L'}$ such that $L' \subseteq L$, $S_L$ is conservative over $S_{L'}$ iff for all $\phi$, $\psi$ in $S_{L'}$ such that $\phi \vdash_{L} \psi$, $\phi \vdash_{L'} \psi$.

Definition 6 (Extension). Given two constrained languages $S_L$ and $S_{L'}$ such that $L' \subseteq L$, $S_L$ extends $S_{L'}$ iff for all $\phi$, $\psi$ in $S_{L'}$ such that $\phi \vdash_{L'} \psi$, $\phi \vdash_{L} \psi$.

Definition 7 (Fully Equality Friendly Language). A constrained language $\langle S_L, \vdash_{L} \rangle$ is fully equality friendly iff for all semantic categories $\alpha$, for all terms $p$, $q$, belonging to $\alpha$, there is a context $e(x)$ such that if $e(p/x) \vdash_{L} e(q/x)$ and $e(q/x) \vdash_{L} e(p/x)$, then for all meaningful contexts $s(x)$ for $p$ and $q$, $s(p/x) \vdash_{LS}(q/x)$.

Before tackling questions 1) to 4), let us make clear the following points. First, traditional logical consequence relations are taken to be structural and monotonic. Here the consequence relation is not purely logical: it might involve special rules for special expressions, therefore structurality does not make sense. As to monotonicity, it might proved hard for a non-monotonic language to be equality friendly in the sense of definition 4, but since equality friendliness is the important thing here, we do not focus on monotonicity. Second, the semantics we are after is not a model-theoretic semantics adequate for the consequence relation. Because we just want to be able to put Hodges (2001) to good use, we shall be satisfied here with the semantics given by a synonymy relation.

To answer question 1), one thus needs assumptions on $\vdash_{L}$ so that it gives a synonymy relation from which a semantics can be constructed in the usual way. We set $\phi \equiv_{\vdash_{L}} \psi$ iff $\phi \vdash_{L} \psi$ and $\psi \vdash_{L} \phi$ and $\lbrack \phi \rbrack \preceq_{\vdash_{L}} \lbrack \psi \rbrack$ if $\phi \vdash_{L} \psi$.

11 Note that Equality Friendliness does not imply Fully Equality Friendliness, it implies it of course for the category of sentences, but not necessarily for other syntactic categories.

12 Other notions of synonymy might be considered, such as having the same consequences, they will generally be provably equivalent only if the consequence relation satisfies certain logical properties.

13 Technically, this is trivial, $\vdash_{L}$ as a relation on sentences will be a preorder and the ordered set of meanings is the order associated to it.
What kind of compositionality do we need for our semantics? As $S_L$ is not closed under subterms, full compositionality certainly does not make sense. What we will need is to satisfy the hypothesis of the first extension theorem, that is $\mu \equiv \vdash_L$ must be 1-compositional in Hodges sense, that is we must have:

If $s(x)$ is a meaningful context for two sentences $\phi$ and $\psi$ such that $\phi \equiv \vdash_L \psi$, then $s(\phi/x) \equiv \vdash_L s(\psi/x)$.

It is trivial to check that $\equiv \vdash_L$ is 1-compositional if and only if $S_L$ is equality friendly.

Question 2) is analogous to Hodges question as to which extension of a semantics from the class of sentences to the class of terms is the good one; in the sense that we need some guiding principles on how to extrapolate from $\vdash_L$ to the semantics for $S_L$.

There are two very different kind of answers to this kind of question:

1. One may wish to restrict the class of available semantics on independent grounds, hoping that this be enough to ensure uniqueness. For example, one might decide that the meaning of a sentence must be a set of possible world.

2. One may wish to add a requirement in the spirit of Hodges full abstraction principle, that is a requirement to the effect that everything that matters to meaning must mirror an explicit inferential link. We will say that the semantics is determinate. Technically, one adds a converse requirement as to what meaning should be: if $\phi \not\equiv \psi$, then $\mu(\phi) \not\leq \mu(\psi)$.

We follow here, by default so to speak, the second strategy: according to it, the algebraic semantics just outlined is the good semantics. First, it is immediate that it satisfies the requirement: if $\phi \not\equiv \psi$, then $\mu(\phi) \not\leq \mu(\psi)$. Second, two semantics satisfying it are unique up to equivalence. Let $\mu$ and $\mu'$ be two such semantics, assume $\phi \equiv \mu \psi$, this means that $\mu(\phi) = \mu(\psi)$, therefore we have both $\mu(\phi) \leq \mu(\psi)$ and $\mu(\psi) \leq \mu(\phi)$, by determinateness, this implies $\mu(\phi) \vdash_L \mu(\psi)$ and $\mu(\psi) \vdash_L \mu(\phi)$, and then by definition of a semantics, $\mu'(\phi) \leq \mu'(\psi)$ and $\mu'(\psi) \leq \mu'(\phi)$, whence $\mu(\phi) = \mu(\psi)$ that is $\phi \equiv \psi$.

We tackle now question 3). Let $\mu$ and $\mu'$ be the two determinate semantics for $S_L$ and $S_{L'}$. $\mu \leq \mu'$ might well be false. Let’s assume that $\phi \equiv \mu \psi$, it might well be the case that $\phi \not\equiv \mu' \psi$, because for example $\phi \vdash_L \psi$ but $\phi \not\vdash_{L'} \psi$. To ensure that this is not the case, it is sufficient that $\vdash_L$ be conservative over $\vdash_{L'}$. If the

\[14\] Another hypotheses is needed for the first extension theorem, husserlianity which says that synonymous expressions belong to the same syntactic type, that is synonymous expressions can be plugged in the same contexts. As we are dealing with sentences, we can assume that this property is always satisfied.
language is rich enough, this is also a necessary condition, assume that \( \phi \vdash_L \psi \)
and \( \phi \not\vdash_L \psi \), under reasonable assumptions on the meaning of \( \land \), we have then
\( \phi \vdash \psi \land \phi \) and of course \( \psi \land \phi \vdash \phi \) so that \( \psi \land \phi \) and \( \phi \) will be \( \mu \)-synonymous but not \( \mu' \)-synonymous.

Note that if determinateness does not hold, it might well be the case that \( \mu \leq \mu' \) though conservativity does not hold, because restrictions on possible meaning assignements were already implementing in \( \mu' \) the synonymies revealed by \( \vdash_L \).

The answer to question 4) is slightly more difficult. If \( \vdash_{LL'} \) is the restriction of \( \vdash_L \) to \( SL' \), as we shall assume from now on, it is immediate that \( \mu' \leq \mu \). But, as we have seen in the proof the proof of the second extension theorem, this will not informative enough when we will be interested in shifting to semantics for \( GL \) instead of \( SL \). We need the separability property: how can we guarantee that it will hold through constraints on \( \vdash \)? The separability property says that if two terms can be separated in the semantics for the big language, they can already be separated in the semantics for the small language. As we have said, it is clear that \( \vdash_L \) being an extension of \( \vdash_{LL'} \) is not in general sufficient to guarantee that. But it will do if there is something in the language like a universal separator for terms of a same semantic category. This is exactly the role the \( e(x) \) contexts in definition 7 of fully equality friendly languages play.\(^{15}\)

Let \( SL \) and \( SL' \) be two fully equality friendly constrained languages, with semantics \( \mu \) and \( \mu' \) such that \( LL' \subseteq L \) and \( SL \) extends \( SL' \). We prove that the separability property holds. Assume for contradiction that there are two terms \( p \) and \( q \) of \( GT(L') \) such that \(^{(*)}p \) and \( q \) can be separated by a context \( s(x) \) in \( SL \), but that there is no separating context for them in \( SL' \). This implies that \( e(x) \) is not such a context, so that \( e(p/x) \) is synonymous with \( e(q/x) \), but then (modulo determinateness) this means that \( e(p/x) \vdash_{LL'} e(q/x) \) and \( e(q/x) \vdash_{LL'} e(p/x) \) hold. By extension, \( e(p/x) \vdash_{LL'} e(q/x) \) and \( e(q/x) \vdash_{LL'} e(p/x) \) hold as well\(^{16}\), so that, by \( SL \) being fully equality friendly \( s(p/x) \vdash_{LS} s(q/x) \) and \( s(q/x) \vdash_{LS} s(p/x) \), which contradicts \(^{(*)} \).

**Dynamic molecularity yields static molecularity**

On the basis of the last subsection, we are now able to define a notion of dynamic molecularity as a property of constrained languages, in a way which fol-

\(^{15}\)The situation here, where the semantics for the big language is constrained by an extension of the consequence relation of the small one, is very different from, and, in fact, better than Hodges purely semantic setting, in which Preservation property has no reason to hold even if the semantics for sentences of the big language is an extension of the semantics for sentences of the small language.

\(^{16}\)We assume here that the \( e(x) \) contexts remain the same in \( SL \) as in \( SL' \).
lows closely the definition of static molecularity.

Let \( L \) be a language constrained by semantic rules \( \vdash_L \) for \( S(L) \). As before, one can imagine two kinds of analysis of \( S(L) \) into simpler languages:

- We can drop a few basic expressions of \( A_L \) in order to see it as the addition of a few new expressions determined in a certain way to an already determined language. This is a shift from \( S(L) \) to \( S(L') \) where \( A_L \supseteq A_{L'} \) and \( A_L \setminus A_{L'} \) is required to be as small as possible. \( \vdash_{L'} \) is be taken to be the restriction of \( \vdash_L \) to sentences of \( S_{L'} \).

- We can see \( S_L \) as the union of already determined smaller languages. This is a shift from \( S_L \) to \( \{ S_{L_i} \}_{i \in I} \), where \( A_L = \bigcup \{ A_{L_i} \}_{i \in I} \). \( \vdash_{L_i} \) are be taken to be the restrictions of \( \vdash_L \) to sentences of \( S_{L_i} \).

**Definition 8 (Molecular IRS Tree).** A molecular tree for a constrained language \( S_L \) is a tree constructed from the root \( S_L \) by applying one of the two operations of analysis just defined, such that at each node, the constrained sub-language is well-behaved and equality friendly, and such that the end nodes are the empty set.

Given a constrained language \( S(L) \), it is clear that if the semantics \( \mu \) for \( GT_L \) is the Fregean extension of the determinate semantics for \( S(L) \), a molecular IRS tree for \( S(L) \) yields, through Hodges first extension theorem, a molecular compositional tree for \( \langle L, \mu \rangle \). Now we just have to capture the analogue of a static molecular analysis. Given a molecular IRS tree for \( S(L) \) labeled with consequence relations \( \vdash_s \) for the constrained sublanguages at each node \( s \), we will say that the property (a) holds at \( s \) iff for every node \( sa \) immediately below \( s \), \( \vdash_s \) is conservative over \( \nu_{sa} \), and that the property (b) holds iff \( \vdash_s \) extends \( \vdash_{sa} \), both of them being also fully equality friendly.

**Definition 9 (Dynamic Molecular Analysis).** A molecular IRS tree for a constrained language \( S(L) \)

- a strong dynamic molecular analysis of \( L \) iff at every node \( s \), properties (a) and (b) hold.
- a cumulative dynamic molecular analysis of \( L \) iff at every node \( s \) property (a) holds.
- an anti-cumulative dynamic molecular analysis of \( L \) iff at every node \( s \) property (b) holds.
- a weak dynamic molecular analysis of \( L \) iff at every node \( s \) either property (a) or property (b) holds.

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\(^{17}\) As long as the consequence relations for parts are restrictions of those for wholes, the first part of (b) is always satisfied.
The last definition is intended to capture the counterpart in term of inferential role of (static) molecularity as a property of a semantics. The following proposition is an easy consequence of the previous subsection:

**Proposition 1.** Given a constrained language \( S(L) \), and a determinate semantics \( \mu \) for \( \text{GT}_L \),

\( \langle L, \mu \rangle \) has a \( X \) static molecular analysis if \( S(L) \) has a \( X \) dynamic molecular analysis, where \( X \) ranges over \{ strong, cumulative, anti-cumulative, weak \}.

5 Conclusion

Our starting point was that (PC) helps explain our mastery of language only because it hints at some sort of epistemic reduction: sentence meanings can be grasped through word meanings, and word meanings can be grasped on a basis which does not involve understanding of all and every sentences of the language. According to this claim, the interesting notion to capture, if one is interested in formalizing some kind of touchstone for theories of languages, is not (PC) alone, but (PC) plus some version of molecularism. Molecularity can be seen as a static property of the semantics of a language, but the interesting point is to understand how it ensues from properties of the basis of our grasp of meanings, i.e. the interesting point is to capture the relevant properties from the point of view of the materials at hand for language learning. This is what section III achieves in the setting of inferential role semantics: we characterize a property of semantic rules of a language which yield the relevant static molecularity for the semantics. A key feature of this analysis is Hodges' work on Fregean extensions, because it gives means to shift from semantic constraints on sentences through semantic rules to semantics for the whole class of terms. Finally, our characterization result is not phrased as concerning one specific notion of molecularity, because we take molecularity to be rather an open range of variably strong properties, the significance of which should be evaluated from both conceptual and empirical perspectives.

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\(^{18}\)We skip the ‘only if’ direction, but the needed provisos have been made clear in the previous subsection as well.
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