Three ways to logicality

Alessandro Giordani
Department of Philosophy - UCSC Milano

Tarski's Thesis
(*Semantic approach to the problem of logicality*).

Formality: logic is formal.
BASIC OPPOSITION: formal *vs* interpreted.
Formal := preserved by all the interpretations.
CRITERION: invariance under all the interpretations.

Generality: logic is general.
BASIC OPPOSITION: general *vs* specific.
General := preserved by all the permutations.
CRITERION: invariance under all the permutations.

1. An Aristotle-Frege variant

(*implementing the formality criterion*).

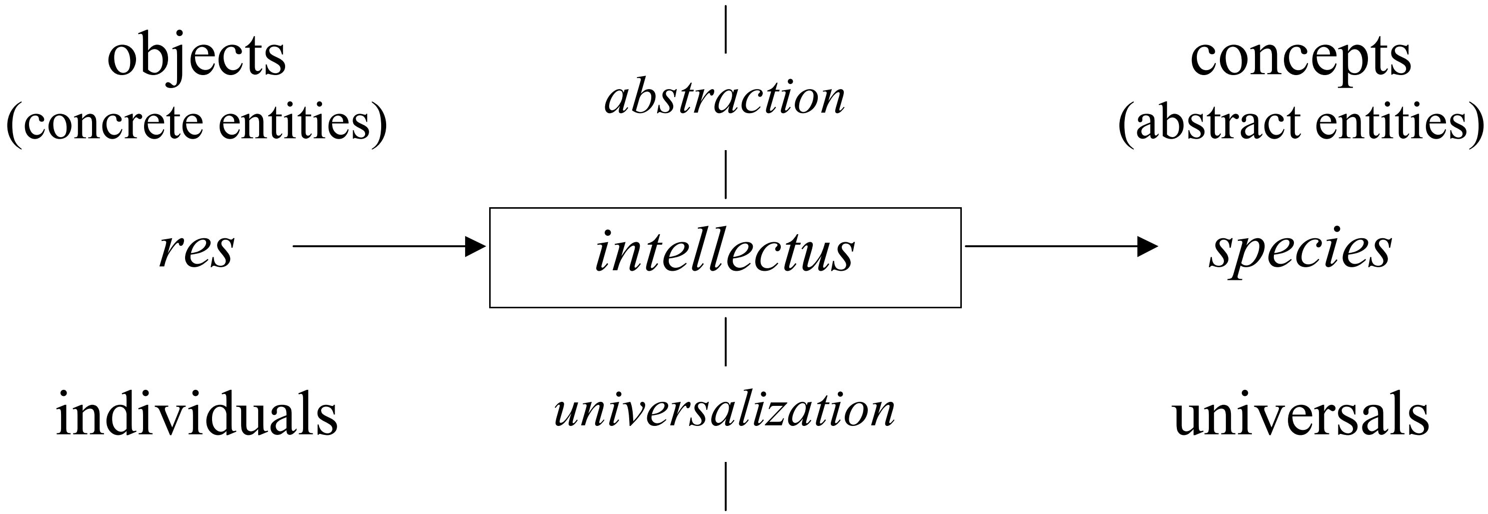
CENTRAL IDEA: logic is the science of the basic laws of the human intellect, to be conceived as the basic laws concerning the realm of entities that are accessible to our intellect.

Identification of logical operators: three steps.

STEP 1: identification of basic logical elements.
STEP 2: identification of basic logical operations.
STEP 3: identification of basic logical operators.

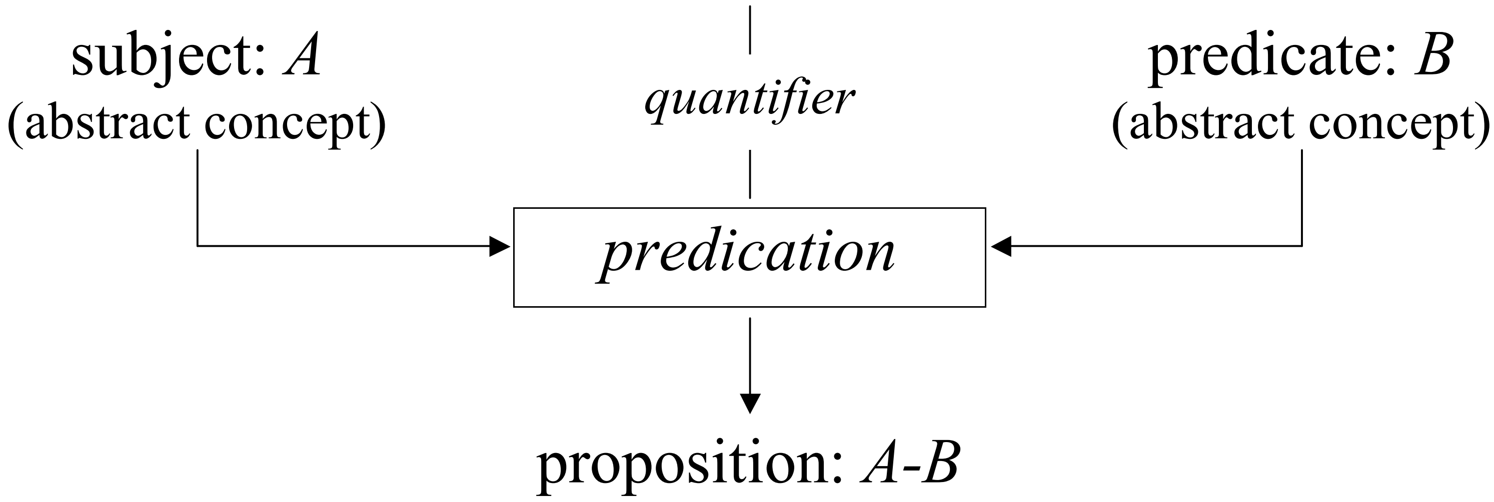
1.1. Aristotle

Step 1: *identification of basic logical elements*.
The first activity of the human intellect is conception.



Step 2: *identification of basic logical operations*.

The second activity of the human intellect is predication.



Step 3: *identification of basic logical operators*.

Logic is formal: the form of predication is independent of the content of the basic logical elements. Hence, the basic logical operators are predication-specifying operators: quantifiers.

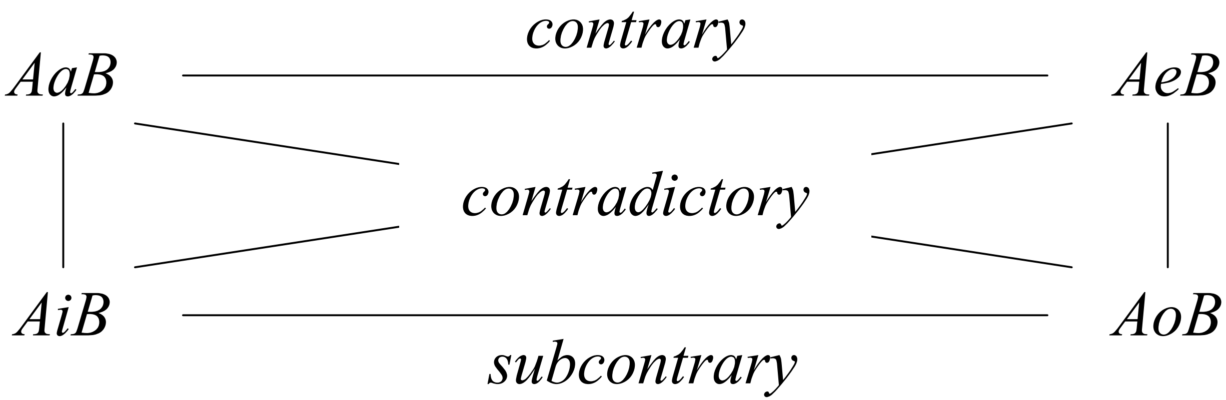
What is an Aristotelian quantifier?

Kinds of predication:

OPPOSITION 1 (*quality*):	positive	*vs*	negative
OPPOSITION 2 (*quantity*):	universal	*vs*	particular

PROPOSITIONS	positive	negative
universal	*AaB*	*AeB*
particular	*AiB*	*AoB*

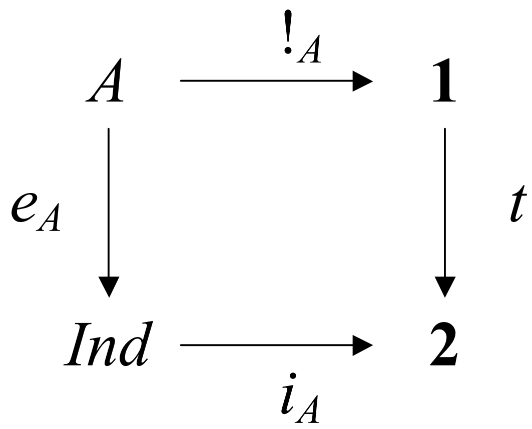
OPPOSITIONS:



Modelling elementary propositions.

1) concepts as species: A, B, . . .
2) concepts as subjects (EXTENSIONS): e_A, e_B, . . .
3) concepts as predicates (INTENSIONS): i_A, i_B, . . .

The relation between the extension and the intension of A:



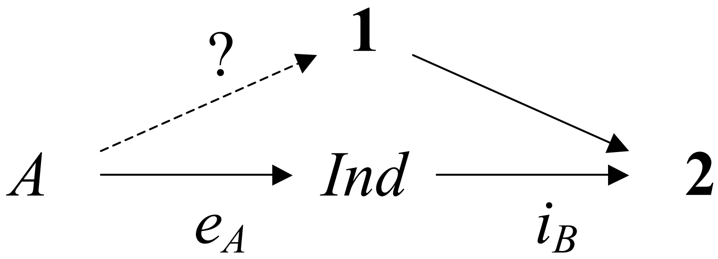
The general form of predication is *A*-*B*:

$$A \xrightarrow[e_A]{} Ind \xrightarrow[i_B]{} \mathbf{2}$$

A-*B* has to be specified with respect to quality and quantity.

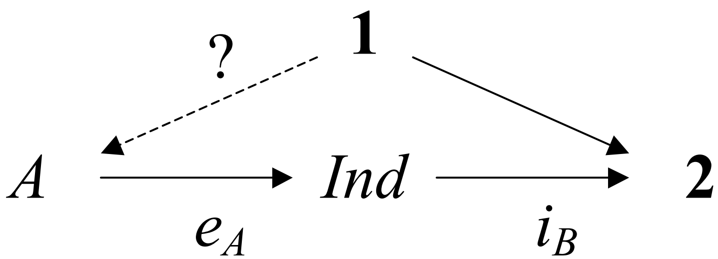
Two choice problems:

Problem 1: Is there a map from *A* to **1** such that



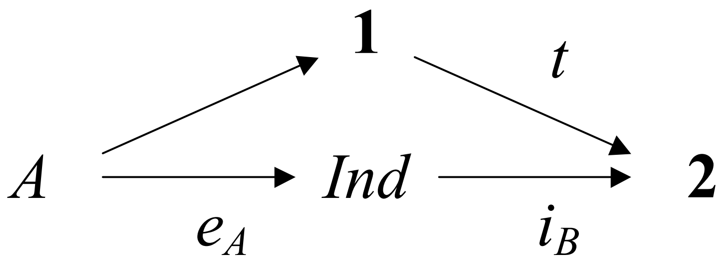
commutes ?

Problem 2: Is there a map from **1** to *A* such that

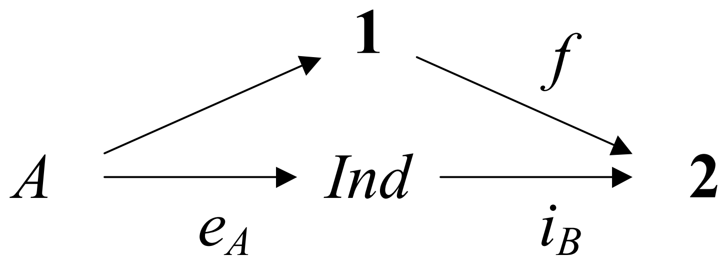


commutes ?

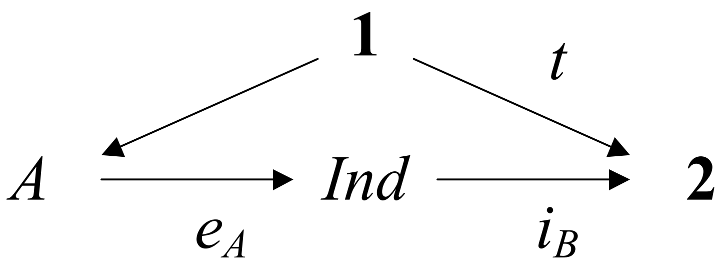
1) AaB:



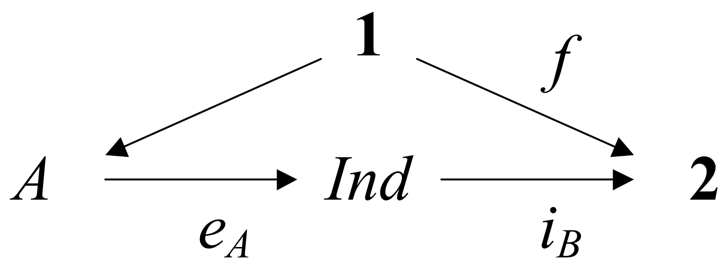
2) AeB:



3) AiB:



4) AoB:



1.2. Frege

Step 1: *identification of basic logical elements*.
1) OBJECTS: essentially non-predicative, saturated entities
2) CONCEPTS: essentially predicative, unsaturated entities.

Step 2: *identification of basic logical operations*.
The basic logical operation is predication, mirroring the onto-logical connection of saturation.

Step 3: *identification of basic logical operators*.
Logic is formal: the form of predication is independent of the content of the basic logical elements.

Duality between concepts and objects

1) concepts as maps from objects to truth values.

$$\mathbf{U}^n \xrightarrow{\quad P^n \quad} \mathbf{2}$$

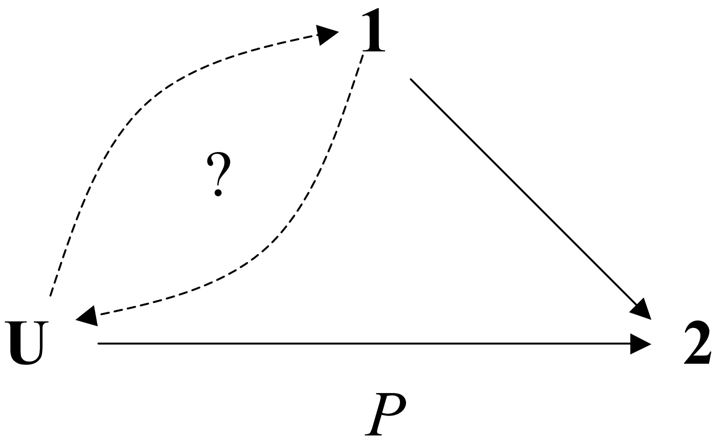
2) objects as maps from concepts to truth values.

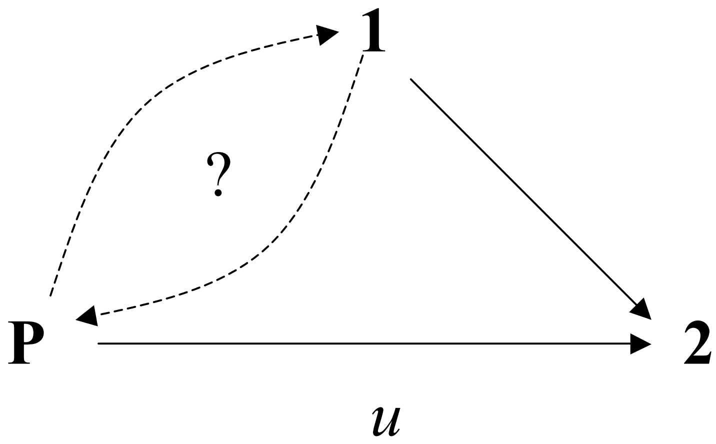
$$\mathbf{P}^n \xrightarrow{\quad u^n \quad} \mathbf{2}$$

3) valuations as maps from concepts × objects to truth values.

$$\mathbf{P}^n \times \mathbf{U}^n \xrightarrow{\quad v \quad} \mathbf{2}$$

Predicative operators as answer to choice problems





universal & existential
1st order quantifiers

universal & existential
2nd order quantifiers

2. A semantic Frege variant

(*implementing the generality criterion*).

CENTRAL IDEA: logic is the science of the basic laws concerning truth, to be conceived as a specific logical object.

Problem 1: a concept *P* is logical <=> . . . ?

$$\mathbf{X} \xrightarrow[P]{\hspace{4cm}} \mathbf{2}$$

P can be considered from two different points of view.

Logicality and constancy

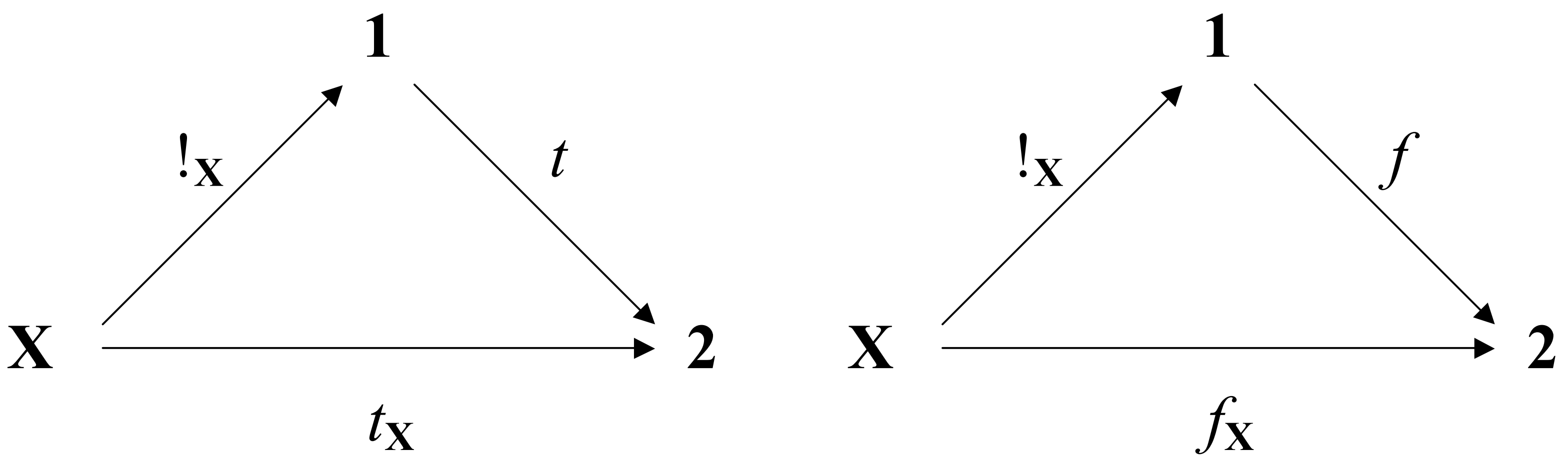
(1) if we focus on the domain of *P*, we acknowledge that a concept defines a sequence of truth values indexed by **X**.

(2) if we focus on the codomain of *P*, we acknowledge that a concept defines a classification of **X** into two classes.

A concept is general if no object can be discerned by using it. Hence, according to (2), one of the classes defined by *P* is **X**; according to (1), the list defined by *P* is a constant list.

Definition: logical concept

A logical concept is a constant sequence of truth values.



A general logical operator can now be introduced as a function that selects logical concepts, i.e. constant sequences in **X**.

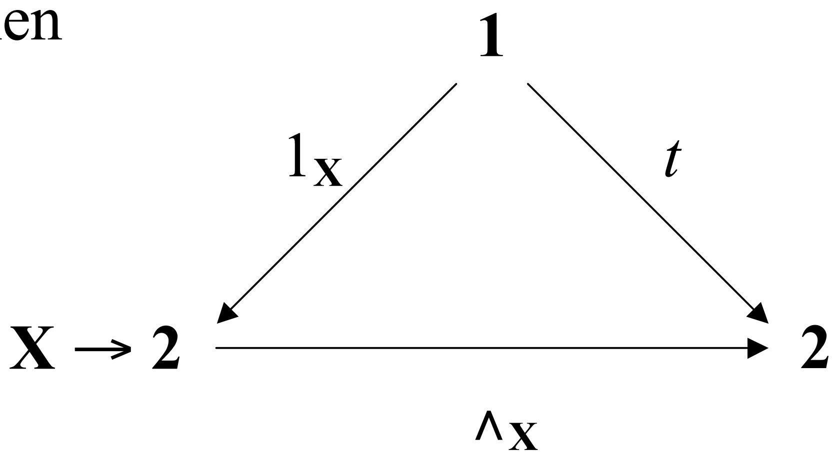
Definition: a logical operator is a selector of logical concepts.

In order to make this definition precise, let

(i) $1_{\mathbf{X}}$: $\mathbf{1} \rightarrow (\mathbf{X} \rightarrow \mathbf{2})$ be the map that selects the constant map $t_{\mathbf{X}} : \mathbf{X} \rightarrow \mathbf{2}$, in $(\mathbf{X} \rightarrow \mathbf{2})$.

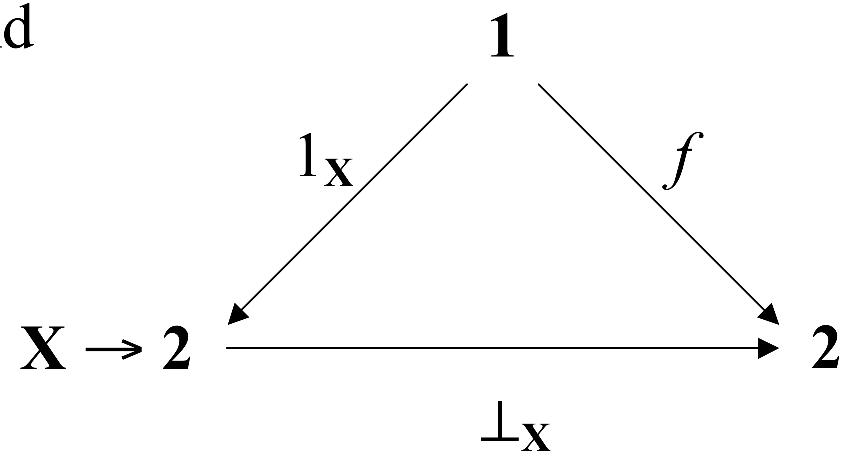
(ii) $0_{\mathbf{X}}$: $\mathbf{1} \rightarrow (\mathbf{X} \rightarrow \mathbf{2})$ be the map that selects the constant map $f_{\mathbf{X}} : \mathbf{X} \rightarrow \mathbf{2}$, in $(\mathbf{X} \rightarrow \mathbf{2})$.

Then



is the general conjunction

and



is the general contrariety

Finally, duality between n-tuples of objects and n-places concepts can be exploited in order to interpret n-tuples of objects as functions from n-places concepts to truth values and take the second order quantifiers as logical operators.

Notice: admitting functions selecting sequences according to any condition preserved by changing the order of 0's and 1's within a sequence is a way to recover TS in its full generality.

3. The in re structuralist variant

(*introducing a criterion of non-eliminability*).

In re, eliminative, structuralism is the view according to which to talk about structures is just a useful way to talk about all systems exemplifying them. To say that it is true of the structure of finite numbers that *A* is tantamount to say that φ is true in any system exemplifying the structure of finite numbers: *places-as-objects* are eliminated in favour of *places-as-offices* and this can be done by quantifying away the constants.

Ex: it is true of the structure of natural numbers that φ is interpreted as

$$\forall X,x,R(PA^X[0/x,S/R] \Rightarrow \varphi^X[0/x,S/R]),$$

where *PA* denotes the conjunction of second order Peano axioms, the superscript denotes relativization of quantifiers to a non-empty domain *X*, and [0/*x*,*S*/*R*] denotes the substitution of the constants denoting zero and the succession relation for appropriate variables.

Assumption: logical constants are structural constants and structural constants are non-eliminable.

But which constants are non-eliminable? The idea is that non-eliminable constants are precisely the ones used in order to perform the elimination, and this provides us with the usual logical constants characterizing second order logic.

That's all