logical constants
defining rules
making it explicit
easy cases
simple modality
2d modality
the upshot
logical constants
what is a logical constant?
it’s logical
it's constant
it’s subject matter independent
it’s definable
it plays a structural role in deduction
logical consequence is necessary
there are paradigm cases,
like $\land$, $\lor$, $\rightarrow$, $\sim$, $\forall$ and $\exists$
what about modality?
boundary drawing
proofs
Tarski’s models, Kripke structures, Montague semantics, etc.

Logical constants are **structure invariant**
Natural deduction, sequent calculus

logical constants have good proof-theoretical rules
proofs

SOUNDNESS

NO OVERLAP

models
proofs

COMPLETENESS

NO GAP

models
proofs and models
proofs and models both play a role in semantics
proofs are good for accounts connected with use...
There are various kinds of inferentialists
Gerhard Gentzen
Die Schlußfiguren-Schemata.

1.21. Schemata für Struktur-Schlußfiguren:

Verdünnung:

im Antezedens: $\frac{\Gamma \rightarrow \Theta}{D, \Gamma \rightarrow \Theta}$, im Sukzedens: $\frac{\Gamma \rightarrow \Theta}{\Gamma \rightarrow \Theta, D}$;

Zusammenziehung:

im Antezedens: $\frac{D, D, \Gamma \rightarrow \Theta}{D, \Gamma \rightarrow \Theta}$, im Sukzedens: $\frac{\Gamma \rightarrow \Theta, D, D}{\Gamma \rightarrow \Theta, D}$;

Vertauschung:

im Antezedens: $\frac{\Lambda, D, E, \Gamma \rightarrow \Theta}{\Lambda, E, D, \Gamma \rightarrow \Theta}$, im Sukzedens: $\frac{\Gamma \rightarrow \Theta, E, D, \Lambda}{\Gamma \rightarrow \Theta, D, E, \Lambda}$;

Schnitt: $\frac{\Gamma \rightarrow \Theta, D}{\Gamma, \Lambda \rightarrow \Theta, \Lambda}$.

1.22. Schemata für Logische-Zeichen-Schlußfiguren:

UES: $\frac{\Gamma \rightarrow \Theta, \mathcal{U}}{\Gamma \rightarrow \Theta, \mathcal{U} \& \mathcal{V}}$  |  OEA: $\frac{\mathcal{U}, \Gamma \rightarrow \Theta, \mathcal{V}, \Gamma \rightarrow \Theta}{\mathcal{U} \vee \mathcal{V}, \Gamma \rightarrow \Theta}$

UEA: $\frac{\mathcal{U}, \Gamma \rightarrow \Theta}{\mathcal{U} \& \mathcal{V}, \Gamma \rightarrow \Theta}$  |  OES: $\frac{\Gamma \rightarrow \Theta, \mathcal{U}}{\Gamma \rightarrow \Theta, \mathcal{U} \vee \mathcal{V}}$  |  EEA: $\frac{\exists x \mathcal{U}, \Gamma \rightarrow \Theta}{\exists x \mathfrak{U}, \Gamma \rightarrow \Theta}$

AES: $\frac{\Gamma \rightarrow \Theta, \mathfrak{U}a}{\Gamma \rightarrow \Theta, \forall x \mathfrak{U}x}$

$\textit{A}$
defining rules
ANALYSIS

THE RUNABOUT INFERENCE-TICKET

By A. N. Prior

It is sometimes alleged that there are inferences whose validity arises solely from the meanings of certain expressions occurring in them. The precise technicalities employed are not important, but let us say that such inferences, if any such there be, are analytically valid.

One sort of inference which is sometimes said to be in this sense analytically valid is the passage from a conjunction to either of its conjuncts, e.g., the inference ‘Grass is green and the sky is blue, therefore grass is green’. The validity of this inference is said to arise solely from the meaning of the word ‘and’. For if we are asked what is the meaning of the word ‘and’, at least in the purely conjunctive sense (as opposed to, e.g., its colloquial use to mean ‘and then’), the answer is said to be completely given by saying that (i) from any pair of statements P and Q we can infer the statement formed by joining P to Q by ‘and’ (which statement we hereafter describe as ‘the statement P-and-Q’), that (ii) from any conjunctive statement P-and-Q we can infer P, and (iii) from P-and-Q we can always infer Q. Anyone who has learnt to perform these inferences knows the meaning of ‘and’, for there is simply nothing more to knowing the meaning of ‘and’ than being able to perform these inferences.

A doubt might be raised as to whether it is really the case that, for any pair of statements P and Q, there is always a statement R such that given P and given Q we can infer R, and given R we can infer P and can also infer Q. But on the view we are considering such a doubt is quite
Inference rules define connectives. 

$\frac{A \quad B}{A \land B} \quad [\land I]$  

$\frac{A \land B}{A} \quad [\land E_1]$  

$\frac{A \land B}{B} \quad [\land E_2]$  

That's all there is to conjunction. You don't need to give truth conditions, satisfaction conditions or any other sort of 'semantics.' These rules tie meaning to use.
But . . . does it work?

\[
\begin{align*}
\frac{A}{A \text{ tonk } B} \quad & \quad \frac{A \text{ tonk } B}{B} \\
\end{align*}
\]

\text{[tonkI]} \quad \text{[tonkE]}
Nuel Belnap
ANALYSIS

TONK, PLONK AND PLINK

By Nuel D. Belnap

A. N. PRIOR has recently discussed the connective tonk, where tonk is defined by specifying the role it plays in inference. Prior characterizes the role of tonk in inference by describing how it behaves as conclusion, and as premiss: (1) A ⊨ T-A-tonk-B, and (2) T-A-tonk-B ⊨ B (where we have used the sign '⊨' for deducibility). We are then led by the transitivity of deducibility to the validity of A ⊨ B, "which promises to banish falsche Spitzfindigkeit from Logic for ever."

A possible moral to be drawn is that connectives cannot be defined in terms of deducibility at all; that, for instance, it is illegitimate to define and as that connective such that (1) A ⊨ A-and-B, (2) A-and-B ⊨ B, and (3) A, B ⊨ A-and-B. We must first, so the moral goes, have a notion of what and means, independently of the role it plays as premiss and as conclusion. Truth-tables are one way of specifying this antecedent meaning; this seems to be the moral drawn by J. T. Stevenson. There are good reasons, however, for defending the legitimacy of defining connections in terms of the roles they play in deductions.

It seems plain that throughout the whole texture of philosophy one can distinguish two modes of explanation: the analytic mode, which tends to explain wholes in terms of parts, and the synthetic mode, which explains parts in terms of the wholes or contexts in which they occur. In logic, the analytic mode would be represented by Aristotle, who commences with terms as the ultimate atoms, explains propositions or judgments by means of these, syllogisms by means of the propositions which go to make them up, and finally ends with the notion of a science as a tissue of syllogisms. The analytic mode is also represented by the contemporary logician who first explains the meaning of complex sentences, by means of truth-tables, as a function of their parts, and then proceeds to give an account of correct inference in terms of the sentences occurring therein. The classical of the application of the synthetic
conjunction is **conservative**, and **uniquely defined** and tonk isn’t

— NUEL BELNAP
this is relative to your starting point
is there an absolute notion?
making it explicit
MAKING IT EXPLICIT

ROBERT B. BRANDON

REASONING, REPRESENTING, & DISCURSIVE COMMITMENT
Inference rules define connectives. For conjunction:

- **\( \land I \)**: \( \frac{A \quad B}{A \land B} \)
- **\( \land E_1 \)**: \( \frac{A \land B}{A} \)
- **\( \land E_2 \)**: \( \frac{A \land B}{B} \)

That's all there is to conjunction. You don't need to give satisfaction conditions or any other sort of 'semantics.' These rules tie meaning to use.
\[\frac{X, A, B \vdash Y}{X, A \land B \vdash Y}[\land L]\]

\[\frac{X \vdash A, Y \quad X' \vdash B, Y'}{X, X' \vdash A \land B, Y, Y'}[\land R]\]
\[ \begin{align*}
X, A, B & \vdash Y \\
\hline
X, A \land B & \vdash Y \\
\end{align*} \quad [\land Df] \]
logical notions have definitions that make explicit in language what is implicit in discourse
this kind of rule provides coordination
\[
\frac{X, A \& B \vdash Y}{X, A, B \vdash Y}[^{\&Df}]
\frac{X, A, B \vdash Y}{X, A \land B \vdash Y}[^{\land Df}]
\]
conservative extension comes too (though this is subtle)
Take a defining rule.

One direction is a LEFT [RIGHT] rule.

Apply the other direction to an identity sequent.

Cut the result against arbitrary sequents, to give the corresponding RIGHT [LEFT] rule.

These LEFT/RIGHT rules allow for the Curry/Belnap Cut-Elimination Argument (at least, when subformulahood is well-founded).

And so, these rules are conservative.
easy cases
∧ ∨ ~ → ∀ ∃
\[
\frac{X, A, B \vdash Y}{X, A \land B \vdash Y} \quad [\land Df]
\]
\[
\frac{X \vdash A, B, Y}{X \vdash A \lor B, Y} \quad [\forall \text{Df}]
\]
\[ X \vdash A, Y \]
\[ \frac{X, \neg A \vdash Y}{X, \neg A \vdash Y} \quad \neg Df \]
\[
\frac{X, A \vdash B, Y}{X \vdash A \supset B, Y} \quad [\supset Df]
\]
\[
\frac{X \vdash A(n), Y}{\frac{X \vdash \forall xA(x), Y}{X \vdash \forall x A(x), Y}} \quad [\forall Df]
\]

(where \( n \) is free in \( X, Y \))
\[ X, A(n) \vdash Y \]

\[ \frac{}{X, \exists x A(x) \vdash Y} \tag{\exists Df} \]

(where \( n \) is free in \( X, Y \))
what do these rules latch on to?
**Abstract:** I argue for the following four theses. (1) Denial is not to be analysed as the assertion of a negation. (2) Given the concepts of assertion and denial, we have the resources to analyse logical consequence as relating arguments with *multiple* premises and *multiple* conclusions. Gentzen’s multiple conclusion calculus can be understood in a straightforward, motivated, non-question-begging way. (3) If a broadly anti-realist or inferentialist justification of a logical system works, it works just as well for *classical* logic as it does for *intuitionistic* logic. The special case for an anti-realist justification of intuitionistic logic over and above a justification of classical logic relies on an unjustified assumption about the shape of proofs. Finally, (4) this picture of logical consequence provides a relatively neutral shared vocabulary which can help us understand and adjudicate debates between proponents of classical and non-classical logics.

---

Our topic is the notion of logical consequence: the link between premises and conclusions, the glue that holds together deductively valid argument. How can we understand this relation between premises and conclusions? It seems that any account begs questions. Painting with very broad brushtrokes, we can sketch the landscape of disagreement like this: “Realists” prefer an analysis of logical consequence in terms of the preservation of *truth* [29]. “Anti-realists” take this to be unhelpful and offer alternative analyses. Some, like Dummett, look to preservation of *warrant to assert* [9, 36]. Others, like Brandom [5], don’t define validity in terms
a **proof** from X to A shows us why it would be a mistake to assert each X and deny A
a **proof** from $X$ to $Y$ shows us why it would be a mistake to assert each $X$ and deny each $Y$.
proofs articulate NORMS governing assertion and denial
a discourse has a score keeping track of what is asserted and denied
defining rules show how we can SCORE **new moves** in the assertion/denial practice, in terms of old moves
Well, yes and so do circuits.

\[ \neg (A \land B) \]

\[ \neg \text{E} \quad \neg \text{I} \]

\[ \bigvee \text{I} \]

\[ \neg A \land B \quad A \land B \]

\[ \neg A \lor \neg B \]

\[ \neg A \lor \neg B \]

\[ \neg \text{I} \]

\[ \neg \text{I} \]

\[ \neg \text{I} \]

\[ \neg \text{I} \]

\[ \neg \text{I} \]

\[ \bigvee \text{I} \]

\[ \neg (A \land B) \]

\[ \neg (A \land B) \]

\[ \neg (A \land B) \]

\[ \neg (A \land B) \]

\[ \neg (A \land B) \]

\[ \neg (A \land B) \]

\[ \neg (A \land B) \]

\[ \neg (A \land B) \]

\begin{align*}
A & \vdash A \\

A & \vdash A \\

A & \vdash A \\

A, \neg A & \vdash A, \neg A \\

A, \neg A & \vdash B, \neg B \\

B, \neg B & \vdash A, \neg A \lor \neg B \\

B, \neg B & \vdash A \land B, \neg A \lor \neg B, \neg A \lor \neg B \\

\neg (A \land B) & \vdash \neg A \lor \neg B, \neg A \lor \neg B \\

\neg (A \land B) & \vdash \neg A \lor \neg B
\end{align*}
Die Schlüssefiguren-Schemata.

1.21. Schemata für Struktur-Schlüssefiguren:

Verdünnung:
im Antezedens:
\[
\frac{\Gamma \rightarrow \Theta}{D, \Gamma \rightarrow \Theta}, \quad \text{im Sukzedens:} \quad \frac{\Gamma \rightarrow \Theta}{\Gamma \rightarrow \Theta, D};
\]

Zusammenziehung:
im Antezedens:
\[
\frac{D, D, \Gamma \rightarrow \Theta}{D, \Gamma \rightarrow \Theta}, \quad \text{im Sukzedens:} \quad \frac{\Gamma \rightarrow \Theta, D, D}{\Gamma \rightarrow \Theta, D};
\]

Vertauschung:
im Antezedens:
\[
\frac{\Lambda, D, E, \Gamma \rightarrow \Theta}{\Lambda, E, D, \Gamma \rightarrow \Theta}, \quad \text{im Sukzedens:} \quad \frac{\Gamma \rightarrow \Theta, E, D, \Lambda}{\Gamma \rightarrow \Theta, D, E, \Lambda};
\]

Schnitt:
\[
\frac{\Gamma \rightarrow \Theta, D \quad D, \Lambda \rightarrow \Theta}{\Gamma, \Lambda \rightarrow \Theta, \Lambda}.
\]

1.22. Schemata für Logische-Zeichen-Schlüssefiguren:

UES: \[
\frac{\Gamma \rightarrow \Theta, \mathrm{U} \quad \Gamma \rightarrow \Theta, \mathrm{V}}{\Gamma \rightarrow \Theta, \mathrm{U} \& \mathrm{V}}.
\]

OEA: \[
\frac{\mathrm{U}, \Gamma \rightarrow \Theta \quad \mathrm{V}, \Gamma \rightarrow \Theta}{\mathrm{U} \lor \mathrm{V}, \Gamma \rightarrow \Theta}.
\]

KEA: \[
\frac{\mathrm{U} \rightarrow \Theta, \Gamma \rightarrow \Theta \quad \mathrm{V}, \Gamma \rightarrow \Theta}{\mathrm{U} \rightarrow \Theta, \mathrm{V}, \Gamma \rightarrow \Theta}.
\]

CSE: \[
\frac{\mathrm{U} \rightarrow \Theta, \mathrm{V} \rightarrow \Theta}{\mathrm{U} \rightarrow \mathrm{V}, \mathrm{V} \rightarrow \Theta}.
\]

AES: \[
\frac{\Gamma \rightarrow \Theta, \exists \mathrm{x}}{\Gamma \rightarrow \Theta, \forall \mathrm{x} \exists \mathrm{x}}.
\]

EEA: \[
\frac{\exists \mathrm{x} \exists \mathrm{x}, \Gamma \rightarrow \Theta}{\exists \mathrm{x} \forall \mathrm{x}, \Gamma \rightarrow \Theta}.
\]
Die Schlußfiguren-Schemata.

1.21. Schemata für Struktur-Schlußfiguren:

Verdünnung:

im Antezedens: \( \frac{\Gamma \rightarrow \Theta}{\mathcal{D}, \Gamma \rightarrow \Theta} \)  
im Sukzedens: \( \frac{\Gamma \rightarrow \Theta}{\Gamma \rightarrow \Theta, \mathcal{D}} \)

Zusammenziehung:

im Antezedens: \( \frac{\Gamma \rightarrow \Theta}{\mathcal{D}, \mathcal{E}, \mathcal{D}, \Gamma \rightarrow \Theta} \)  
im Sukzedens: \( \frac{\Gamma \rightarrow \Theta}{\Gamma \rightarrow \Theta, \mathcal{D}, \mathcal{E}, \mathcal{A}} \)

Vertauschung:

im Antezedens: \( \frac{\Gamma \rightarrow \Theta}{\mathcal{D}, \mathcal{A}, \mathcal{D}, \Gamma \rightarrow \Theta} \)  
im Sukzedens: \( \frac{\Gamma \rightarrow \Theta}{\Gamma \rightarrow \Theta, \mathcal{D}, \mathcal{E}, \mathcal{A}} \)

Schnitt: \( \frac{\Gamma \rightarrow \Theta, \mathcal{D}}{\Gamma, \mathcal{A}, \Gamma \rightarrow \Theta} \)

connective rules make implicit aspects of the scoresheet explicit

1.22. Schemata für Logische-Zeichen-Schlußfiguren:

UES: \( \frac{\Gamma \rightarrow \Theta, \mathcal{A}}{\Gamma \rightarrow \Theta, \mathcal{A}, \mathcal{B}} \)  

UES: \( \frac{\mathcal{A}, \Gamma \rightarrow \Theta}{\mathcal{A} \& \mathcal{B}, \Gamma \rightarrow \Theta} \)  

AES: \( \frac{\Gamma \rightarrow \Theta, \exists \alpha}{\Gamma \rightarrow \Theta, \forall \alpha \exists \alpha} \)

OEA: \( \frac{\mathcal{A}, \Gamma \rightarrow \Theta}{\mathcal{A} \lor \mathcal{B}, \Gamma \rightarrow \Theta} \)

OES: \( \frac{\Gamma \rightarrow \Theta, \mathcal{A}}{\Gamma \rightarrow \Theta, \mathcal{A} \lor \mathcal{B}} \)

EEA: \( \frac{\exists \alpha, \Gamma \rightarrow \Theta}{\exists \alpha \exists \alpha \exists \alpha, \Gamma \rightarrow \Theta} \)
this connects semantics with normative pragmatics
THIS logic/non-logic boundary is determined by **two choices**
the structural context, given by the space of scores
and the choice of vocabulary, given that context
simple modality
a **proof** from $X$ to $Y$ shows us how a position in which each $X$ is asserted and each $Y$ is denied is OUT OF BOUNDS
assertion and denial needn’t be flat
I can assert or deny under a supposition
an assertion of

“I’m in Ljubljana”

clashes with its denial.
an assertion of
“I’m in Ljubljana”
doesn’t clash with denying
“I’m in Ljubljana” under the
scope of “suppose I stayed
home to teach my classes”
modal discourse
is filled with shifts like these
why not take this into account in **scoring** discourse?
\( X \vdash Y \) tells us that it’d be a mistake to \textcolor{green}{assert} \( X \) and \textcolor{red}{deny} \( Y \)
\[ X \vdash Y \mid U \vdash V \] tells us that it’d be a mistake to **assert** X and **deny** Y (in one part of the discourse) and to **assert** U and **deny** V (in another).
hypersequents suit proof systems for modal logics
\[
\frac{\mathcal{H}[X \vdash \Box A, Y]}{\mathcal{H}[\vdash A \mid X \vdash Y]} \quad [\Box Df]
\]
Greg Restall, that if is derivable in a derivation of exactly the same length. A hypersequent, with extra formlulas added in the left or right of a component alternative sequent and to the sequent, and so does these complete the rules of the sequent system.

For the induction step, notice that in each rule use to derive:

\[
\frac{\mathcal{H}[X \vdash Y \mid X', A \vdash Y']}{\mathcal{H}[X, \Box A \vdash Y \mid X' \vdash Y']} [\Box L]
\]

\[
\frac{\mathcal{H}[\vdash A \mid X \vdash Y]}{\mathcal{H}[X \vdash \Box A, Y]} [\Box R]
\]

As examples:

http://consequently.org/writing/cfss2dml/
\[
\begin{align*}
\frac{A \vdash A}{\Box A \vdash \vdash A} \quad [\Box L] \\
\frac{B \vdash B}{\Box B \vdash \vdash B} \quad [\Box L] \\
\frac{\Box A, \Box B \vdash \vdash A \land B}{\Box A \land \Box B \vdash \vdash A \land B} \quad [\land R] \\
\frac{\Box A \land \Box B \vdash \vdash A \land B}{\Box A \land \Box B \vdash \Box (A \land B)} \quad [\Box R]
\end{align*}
\]
\[ \mathcal{H}[X \vdash Y \mid X', A \vdash @ Y'] \]

\[ \mathcal{H}[X, @A \vdash Y \mid X' \vdash @ Y'] \] [:@Df]
2d modality
there are TWO DIFFERENT KINDS of shift
indicative
(suppose I’m wrong)
and subjunctive
(suppose things go differently)
suppose Oswald didn’t shoot JFK

suppose Oswald hadn’t shot JFK
What is a person, as opposed to a non-person? One might begin to address the question by appealing to a second distinction: between agents, characterized by the ability to act freely and intentionally, and mere patients, caught up in events but in no sense authors of the happenings involving them. An alternative way to address the question appeals to a third distinction: between subjects — bearers of rights and responsibilities, commitments and entitlements, makers of claims, thinkers of thoughts, issuers of orders, and posers of questions — and mere objects, graspable or evaluable by subjects but not themselves graspers or evaluators.

We take it as a methodological point of departure that these three distinctions are largely coextensive, indeed coextensive in conceptually central cases. Granted, these distinctions can come apart. One might think that ‘person’ applies to anything that is worthy of a distinctive sort of moral respect and think this applicable to some fetuses or the deeply infirm elderly. Even if the particular respect due such beings is importantly different from “what we owe each other”, such respect could still be thought to be of the kind distinctively due people, and think this even while holding that such people lack agentive or subjective capacity. Similarly, one might think dogs or various severely impaired humans to be attenuated subjects but not agents.

Without taking any particular stand on such examples, our methodological hypothesis is that such cases, if they exist, are understood as persons (agents, subjects) essentially by reference to paradigm cases and, indeed, to a single paradigm within which person/non-person, subject/object, and agent/patient are conceptually connected.1 Stated

1. For one detailed development of this sort of paradigm-riff structure, and a defense of the possibility of concepts essentially governed by such a structure, see Lance and Little (2004). Discussions with Hilda Lindeman have helped
indicative and subjunctive shifts are independently motivated for creatures who act on the basis of their views
this structure grounds a system for a 2D modal logic for **necessity** (**SUBJUNCTIVE**) a priori knowability (**INDICATIVE**) & **actuality** (interacts with **BOTH**)
\[
\frac{a = b \vdash \quad Fa \vdash Fb}{a = b \vdash \quad \top Fa \supset Fb} \quad [\top Df]
\]
\[
\frac{a = b \vdash \quad \top Fa \supset Fb}{a = b \vdash \quad \Box (Fa \supset Fb)} \quad [\Box Df]
\]

\[
\frac{a = b \vdash \quad || Fa \vdash Fb}{a = b \vdash \quad || \vdash Fa \supset Fb} \quad [\top Df]
\]
\[
\frac{a = b \vdash \quad || \vdash Fa \supset Fb}{a = b \vdash \quad \Box K (Fa \supset Fb)} \quad [\Box K Df]
\]

**good**  
**bad**
\[
\begin{align*}
\frac{a = b \vdash \, | \, Fa \vdash Fb}{a = b \vdash \, | \, \top \vdash Fa \supset Fb} \quad & [\top Df] \\
\frac{a = b \vdash \, | \, \top \vdash Fa \supset Fb}{a = b \vdash \Box (Fa \supset Fb)} \quad & [\Box Df]
\end{align*}
\]

good

\[
\begin{align*}
\frac{a = b \vdash | \top \vdash Fa \supset Fb}{a = b \vdash | \top \vdash Fa \supset Fb} \quad & [\top Df] \\
\frac{a = b \vdash | \top \vdash Fa \supset Fb}{a = b \vdash \text{APK} (Fa \supset Fb)} \quad & [\text{APK} Df]
\end{align*}
\]

bad
\[
\frac{\vdash p \vdash @ p}{\vdash p \vdash @ @ p} \quad [@Df] \\
\frac{\vdash @ p \vdash @ p}{\vdash @ p \vdash @ p} \quad [\sqcap Df] \\
\frac{\vdash @ p \vdash @ p}{\vdash @ p \vdash @ p} \quad [\square Df] \\
\frac{\vdash @ p \vdash @ p}{\vdash @ p \vdash @ p} \quad [\sqcup Df] \\
\frac{\vdash APK (p \vdash @ p)}{\vdash APK (p \vdash @ p)} \quad [APK Df]
\]

bad

good
\[
\begin{align*}
\vdash & \quad \varphi \vdash \Box \varphi \\
\vdash & \quad \varphi \vdash @\varphi \\
\vdash & \quad \varphi \vdash @\Box \varphi \\
\vdash & \quad \Box (\varphi \vdash \Box \varphi) \\
\end{align*}
\]

bad

\[
\begin{align*}
\vdash & \quad \varphi \vdash \Box \varphi \\
\vdash & \quad \varphi \vdash @\varphi \\
\vdash & \quad \varphi \vdash \Box \varphi \\
\vdash & \quad \Box (\varphi \vdash \Box \varphi) \\
\end{align*}
\]

good
A CUT-FREE SEQUENT SYSTEM FOR
TWO-DIMENSIONAL MODAL LOGIC,
AND WHY IT MATTERS

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Abstract: The two-dimensional modal logic of Davies and Humberstone [3] is an important aid to our understanding the relationship between actuality, necessity and a priori knowability. I show how a cut-free hypersequent calculus for 2D modal logic not only captures the logic precisely, but may be used to address issues in the epistemology and metaphysics of our modal concepts. I will explain how use of our concepts motivates the inference rules of the sequent calculus, and then show that the completeness of the calculus for Davies–Humberstone models explains why those concepts have the structure described by those models. The result is yet another application of the completeness theorem.

MOTIVATION

The ‘two-dimensional modal logic’ of Davies and Humberstone [3] is an important aid to our understanding the relationship between actuality, necessity and

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the upshot
these rules are conservative and uniquely defining
if we agree on what indicative and subjunctive shifts occur in a discourse then we coordinate on these modal concepts
(we can coordinate on the meaning of □ without agreeing on whether or not a particular necessity claim is true)
(after all, we can coordinate on the meaning of $\land$ without agreeing on whether or not a particular conjunction is true)
these modal concepts arise freely from the STRATIFIED structure of our discourse
the rules show how these modal concepts are grounded in our CAPACITIES
the rules tell us how to reason
with these modal concepts
and so, can play a role in modal epistemology
the general structure of completeness theorems gives us something to say about possible worlds, too
and so, this can play a role in modal ontology
thank you!
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