

LOGICAL CONSTANTS, adding some angles

ESSLLI Workshop, Ljubljana 8 August 2011

Johan van Benthem, <http://staff.science.uva.nl/~johan>

1 Some approaches to logicity

* Permutation invariance, top end of transformation/invariance view. * Proof patterns, hinges of inference. * Computation, procedure control, communication, multi-agent games. Different ideas: logic as *world description*, logic as *discourse structuring*. Not one notion? Today: inject some ideas from less standard semantics. Sources of variation or extension.

2 Invariance over a given signature: changing the transformations

Function words. Are prepositions logical? Indexicals? Mass quantifiers? Modalities: Knowledge $K\varphi$, counterfactuals $\varphi \Rightarrow \psi$: logical, or not? Invariants for automorphisms of basic structure (accessibility, similarity). ‘Logical’ over natural signatures? ($K(R, \varphi)$)

3 Aside: existing ideas extend to dynamic modalities of model change

Actions of model change: public announcement $!\varphi$, radical upgrade $\uparrow\varphi$, questions $?\varphi$. These operators are permutation invariant: isomorphic inputs give isomorphic values. Further constraints needed, like in classical case. New issue: where best locate the logic. *DEL* product update: one logical mechanism $[E, e]\varphi$, dropping a lot of separate operators.

4 Changing the model class

Logical constants change when we change the model. Example: first-order logic over abstract assignment models, dropping the ‘gluing property’ of full assignment spaces. Logical constants are the usual ones (viewed as modalities, and bisimulation invariant), but *new ones arise*, such as polyadic quantifiers $\exists x$ over tuples of objects. Safety over p.i.’s?

5 Changing the arguments of logical operators

Propositions as evaluation procedures, transition relations between assignments, dynamic predicate logic *DPL*. Basic logical constants dynamic \sim (test negation), $;$ (sequential composition). Surprise: quantifiers $\exists x$ not logical operators, but *atomic actions* of value resetting for x ! Core algebra of $\{\sim, ;\}$ over object picking and fact testing is *decidable*.

6 Logicality, computation, and procedure

Syntax carries richer information than denotations. Logicality as basic ‘procedural control’. Non-first-orderness equally natural then, e.g., Kleene iteration of actions. Make it explicit. Challenge: in which sense are fixed-point operators logical? Should logical constants be computationally simple? What mathematical framework best suited: automata theory?

7 Propositions as games

Game-theoretic semantics for first-order logic. Natural logical operations are now at least: ^d dual (role switch), \wedge , \vee (choices for players), $;$ (composition). Again, $\exists x$ is not a game operation, but an atomic game. Challenge: what is the right notion of game equivalence? Extensional forcing powers, more intensional structures? Need better theory of interaction. Much wider variety of natural game operations: kinds of parallel composition \parallel , analogies with game semantics and process algebra. Dominant approach: logicality as respect for game equivalence. Another approach: logical constants as natural hinges of proof theory.

8 Conclusion

Descriptive versus operative aspect. From world description to process function once more. Different views on logicality may be the ‘natural stances’ of logic.

9 References

- J. van Benthem, 1996, *Exploring Logical Dynamics*, CSLI Publications, Stanford.
- , 2002, ‘Invariance and Definability: two faces of logical constants’, in W. Sieg, R. Sommer & C. Talcott, eds., *Reflections on the Foundations of Mathematics. Essays in Honor of Solomon Feferman*, ASL Lecture Notes in Logic 15, 426–446.
- , 2003, ‘Logic Games are Complete for Game Logics’, *Studia Logica* 75, 183–203.
- , 2008, ‘Four Intuitions of Logicality, and still counting, Lecture, IHPRST, Paris.
- , 2011, *Logical Dynamics of Information and Interaction*, Cambridge UP.
- , to appear, *Logic in Games*, Texts in Logic and Games, Springer Verlag.
- J. van Benthem & D. Bonnay, 2008, ‘Modal Logic and Invariance’, *Journal of Applied Non-Classical Logics*, 18:2-3, 153 – 173.