

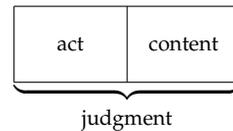
# Logic without constants

Max Weiss

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## 0.1 Thesis statement

1. Are there any mountains in Slovenia that are over 3000m tall?
  - (a) Suppose that one person answers yes, another no.
2. The two answers express judgments that are in conflict.
  - (a) The conflict points to a difference between one and the other.
  - (b) But what is the basis of this difference?
3. Frege and Wittgenstein agree that (in the simplest case) a judgment can be represented as an act on a content, like so:



4. With respect to this schema, we can represent by shading where Frege and Wittgenstein respectively locate the difference that underlies the given disagreement:



5. Thus, Frege and Wittgenstein disagree about nature of the difference between "yes" and "no".
6. This disagreement about the basis of contradiction generalizes to a disagreement about the basis of all logical relationships.
  - (a) The generalization follows from a reduction claimed (however implausibly) by Wittgenstein.

## 0.2 Interpretive context

1. I don't want to get into secondary literature in this talk, but let me just indicate how the thesis relates to the literature.
  - (a) Broadly, my attributions to Frege are intended to be uncontroversial.
  - (b) The reading of Wittgenstein conflicts e.g. with this claim of Ricketts (1985), that "Wittgenstein shares Frege's view of judgment."
    - i. That claim has got to be incompatible with my thesis.
    - ii. The basis of the incompatibility will be in a difference over our respective interpretations of Wittgenstein.

## 0.3 Precursor situation

1. According to my thesis, Wittgenstein stands to Frege as Kant stands to Bolzano.
  - (a) Bolzano's logic is based around the notion of the *Satz-an-sich*, or proposition-in-itself.
    - i. The terminology is a jab at Kant.
2. For Kant, a judgment is an act of synthesis.
  - (a) The output is a compound representation, e.g., that "some  $F$  is not  $G$ ," and the inputs are representations  $F$  and  $G$ .
  - (b) Synthesis determines the form of the compound representation.
    - i. It selects one of three options from each of four menus: quantity, quality, relation, and modality.
    - ii. The first three menus (roughly) give choice of quantifier, placement of negation, and options for disjunction and conditional.
    - iii. The fourth menu modifies the copula, according as the output representation is put forth as true of necessity, or as contingently, or on the other hand, not put forth as true but rather only as a hypothesis.
  - (c) So for Kant, what now look like matters of logical form get resolved alongside whether the compound representation is accepted as true or instead merely contemplated.
  - (d) In this sense, it's judgment that determines logical form, so we can say:
    - i. For Kant, logical forms are forms of judgment.
3. I claim that the same is true for Wittgenstein: logical forms are forms of judgment.
4. This is in contrast to Frege, for whom logical form characterizes rather the contents acknowledged by judgment.

# 1 Frege on judgment

1. For Frege, judgment is the recognition of truth of a thought.
2. But a thought can be grasped without recognizing it as true,
3. and it exists independently of our grasping it, like a hammer.
4. A thought is complete in itself and exists self-standingly.
  - (a) We no more create a thought in thinking it than a traveller creates a mountain range in crossing it (*dV*, 151).
5. Nonetheless: somebody can ask after the truth of a thought never before grasped by a terrestrial being, and other people will understand.
  - (a) The compositeness of the question's verbal expression allows us to recognize the thought by a corresponding composition of its parts.
  - (b) Some parts of the thought are considered "complete" some "incomplete."
    - i. For example, the thought  $\neg Fa$  will have the complete part  $Fa$  and the incomplete part  $\neg\xi$ .
    - ii. In turn  $Fa$  has the complete part  $a$  and the incomplete part  $F\xi$ .
6. Logical constituents are distinguished by the fact that their properties make demands on all thinking (rather than just on thinking about e.g. physical space).
7. But with respect to the structure of the contents of judgment, nothing distinguishes a logical constituent like  $\neg\xi$  from a nonlogical constituent  $F\xi$ : each is a function on the domain of complete entities.

# 2 The basic argument

1. Here is the basis of Wittgenstein's disagreement with the Fregean view just described.
  - (1) A proposition is truth-function of elementary propositions.
  - (2) Truth-functions are iterations of Verneinung.
  - (3) Verneinung is (generalized joint) denial.
  - (4) Denial is a form of judgment.
  - (5) So a proposition is the iteration of a form of judgment over the elementary propositions.
2. I'll discuss the premises in reverse order.

# 2.1 Denial is a mode of judgment

1. I want to understand how W confronts Frege with the claim that there is such a phenomenon, denial as a mode of judgment, so the point of this section is to cast the notion of denial in Frege's terms.
2. The yes-no motif of §0.1 is due to Frege's 1919 paper *Die Verneinung*.
  - (a) For Frege, "the very nature of a question demands a separation between the acts of grasping a sense and of judging" (*dV*, 145).
3. However, the practice also suggest that there two forms of judging, the affirmative and the negative.
4. Frege objects (*dV*, 152) that this view obscures the validity of the following pair of arguments:

|  |  |
|--|--|
| (A1) If the accused was in Rome,<br>then he is innocent. | (B1) If the accused was not in Berlin,<br>then he is innocent. |
| (A2) Was the accused in Rome?<br>—Yes.                   | (B2) Was the accused in Berlin?<br>—No.                        |
| (A3) So he is innocent.                                  | (B3) So he is innocent.  |
5. To Frege's proponent of denial, the two arguments would be represented as follows:

|                             |                                  |
|-----------------------------|----------------------------------|
| (A1') Aff $p \rightarrow r$ | (B1') Aff $\neg q \rightarrow r$ |
| (A2') Aff $p$               | (B2') Den $q$                    |
| (A3') Aff $r$ 1,2(MP)       | (B3') Aff $r$ 1,2(???)           |
6. In contrast, Frege would represent the premise B2 as Aff  $\neg r$  so that second argument would be valid in precisely the same pattern as the first.
7. Since Frege's account uses fewer resources, he concludes that we should reject the doctrine that affirmation and denial are parallel species of judging, separate negation from the act of judging, and instead "unite negation with the thought."
8. From this objection of Frege's, we can extract the following characterization of the notion he rejects:
  - I. Denial is coordinate with affirmation.
  - II. The content of a denial of  $p$  is the content of an affirmation of  $p$ .

## 2.2 Verneinung is denial

**I. Verneinung is coordinate with Bejahung.** This can be seen in the table of occurrences of “Beja-/beja-” and “Vernein-/vernein-.”

### Occurrences:

|       |     |  |  |
|-------|-----|--|--|
| 342   | v   |  |  |
| 40621 | v   | <b>Examples:</b>   |  |
| 4064  | b v | 6231: It is a property of affirmation that one can produce it by double negation.  |  |
| 40641 | v   | <i>Es ist eine Eigenschaft der Bejahung, dass man sie als doppelte Verneinung auffassen kann.</i>  |  |
| 5124  | b   |  |  |
| 51241 | b v |  |  |
| 52341 | v   | 4064: Each proposition must already have a sense; affirmation cannot give it a sense, for its sense is just what it is affirmed. And the same holds for negation, etc. |  |
| 5254  | v   |  |  |
| 544   | b v | <i>Jeder Satz muss schon einen Sinn haben; die Bejahung kann ihn ihm nicht geben, denn sie bejaht ja gerade den Sinn. Und dasselbe gilt von der Verneinung, etc.</i>   |  |
| 5451  | v   |  |  |
| 55    | v   |  |  |
| 5512  | v   |  |  |
| 5513  | b   |  |  |
| 5514  | b v |  |  |
| 6231  | b v |  |  |

### II. The content of a Verneinung of $p$ is the content of a Bejahung of $p$ .

1. This fails trivially, if consistency/consequence/etc supervene on content.
2. However, there is a stable, central notion of content in the *Tractatus* according to which the property (II) does hold.
  - (a) We find it explicitly in some pre-Tractarian writings.
  - (b) It’s an adaptation of Frege’s distinction of sense and reference, specialized to propositions as Wittgenstein then understood them.
3. In the “Notes on logic” (1913) and “Moore notes” (1914), he proposes that a proposition has both a “sense/Sinn” and a “meaning/Bedeutung.”
  - (a) The “sense” of a proposition is its truth-condition (95).
  - (b) One obtains the “Bedeutung” of a proposition as follows.
    - i. To each elementary proposition there corresponds a state of affairs whose existence would make it true; its Bedeutung is the existence of that state of affairs if it exists and is the nonexistence of that state of affairs otherwise.
    - ii. For each truth-function  $A$  of elementary propositions, an elementary proposition  $p$  belongs to its basis if the truth-value of  $A$  depends on the truth-value of  $p$ .

iii. The *Bedeutung* of  $A$  consists of the *Bedeutungen* of the elements of the basis of  $A$  (93, 94, 111).

(c) Thus,  $p$  has the same meaning as  $\neg p$  (94); they have “the same meaning but opposite sense” (97).

4. The terminology (understandably) disappears in the *Tractatus*; W now says that only names have *Bedeutung* and propositions *Sinn* (6.124).
5. But *Bedeutung* of a proposition becomes “what corresponds to the proposition in reality:”
 

The propositions “ $p$ ” and “ $\neg p$ ” have opposed sense, but there corresponds to them one and the same reality (4.0621).
6. The persistence of *Satzbedeutung* underlies W’s thesis that “a proposition is a truth-function of elementary propositions” (T5).
  - (a) W knew Frege’s writings, and he was a student of Russell: so this is a seemingly unforced move that needs explanation.
  - (b) I claim that W’s demand for the general applicability of *Satzbedeutung* precludes Fregean treatment of the quantifiers.
7. For Frege, the sense of  $\forall xFx$  is independent of the contents of the domain.
 

“ $\text{e. } \Phi(a)$ ” is to denote the True if for every argument the value of the function  $\Phi(\xi)$  is the True, and otherwise is to denote the False (Gg, §8).
8. Wittgenstein instead attempts a truth-functional analysis of e.g.  $\forall xFx$ .
  - (a) According to 5.5 and 5.502, each truth-function is a result of successive applications to elementary propositions of  $N(\dots)$ 
    - i. the brackets contain an expression for a plurality of propositions
    - ii. and  $N$  abbreviates the column of the truth-table for a joint negation of appropriate arity.
  - (b) 5.501 says that the propositional arguments of the joint negation might be specified in any of at least the following three ways: by explicit enumeration, or, by presenting them as values of a function  $fx$  for all values of  $x$ , or by means of some formal rule of construction.
    - i. And he furthermore says “how you describe the arguments is inessential” (5.501);
    - ii. “inessential” means “irrelevant to sense,” because propositions are individuated by their senses (cf. esp. 334, 331, 3317, 4013, 44661).
  - (c) E.g., supposing the arguments to be specified by plural term  $\bar{\xi}$  whose values are  $p$  and  $q$ , then:  $N(\bar{\xi})$ , and  $N(p, q)$ , and  $\neg p \wedge \neg q$ , are all the same proposition (5.51).

9. W doesn't explicitly reduce his quantifier notation to this framework, but:
- (a) write  $\{\Phi(x)\}_x$  for the plurality of propositions that result from  $\Phi(x)$  by assigning a value to  $x$ ; then  $\forall xfx$  becomes  $N(\{N(fx)\}_x)$ .
  - (b) This is not genuine quantification:
    - i. the identity of the proposition  $N(\{N(fx)\}_x)$  turns on which propositions occur in the plurality  $\{N(fx)\}_x$ , hence on which objects occur in the range of  $x$ .
    - ii. The first  $x$  in  $\{N(fx)\}_x$  is not genuinely bound; the device is like abbreviating  $p_1 \vee \dots \vee p_7$  as  $\bigvee_{i \leq 7} p_i$ .
  - (c) Contrasting explicitly the Fregean and Wittgensteinian evaluation clauses:
    - (F)  $\forall x\Phi(x)$  is the True iff  $\Phi(\xi)$  is the True for every argument.
    - (W)  $\forall x\Phi(x)$  is true iff  $\Phi(a)$  is true and  $\Phi(b)$  is true and  $\dots$
  - (d) The *Tractatus* does not give any recipe for genuine quantification but only for sugaring of truth tables.
    - i. The confusion is masked by fixed-domain semantics.
    - ii. It probably underlies the notorious claim that quantificational validity is decidable (6113, 6126).

### 2.3 Truth-functions are iterations of Verneinung

1. Wittgenstein's basic description of the totality of propositions is analogous to the iterative conception of set.
  - (a) One begins with the totality of elementary propositions.
  - (b) Now, one inductively applies the following step: as many times as possible, collect some already-given propositions  $\bar{p}$  and form the proposition  $N(\bar{p})$ .
2. So it looks like the iterated application of this  $N$  operation is essential to Wittgenstein's conception of the proposition.
3. I've argued, though, that  $N$  expresses denial, a mode of judgment.
4. So the view I attribute to Wittgenstein conflicts with a celebrated doctrine of Frege.
  - (a) The doctrine has two versions:
    - i. First, there is a linguistic version, which is that the force-modifier can't be embedded in other operators.
    - ii. Second there is the judgment-theoretic version, that modification of judgment govern the whole content.

- (b) The linguistic version of the doctrine is widely accepted (cf. Dummett 1981, 285ff).
  - (c) But, I think that the linguistic version of Frege's thesis doesn't directly apply in the present situation.
    - i. Propositions for Wittgenstein don't have normal syntactical structure:
    - ii. For example, it's not a well-defined question whether  $\neg$  has an embedded occurrence in  $\neg\neg p$ , since Wittgenstein holds that  $\neg\neg p$  is the same proposition as  $p$ .
  - (d) So we need to consider directly the judgment-theoretic version of the thesis.
  - (e) This is the version that puts Frege in conflict with Kant, and similarly, I claim, with Wittgenstein.
5. There is even a prima facie difficulty in the account I've offered:
    - (a) How can a denial of a denial of  $p$  amount to an affirmation of  $p$ ?
  6. Wittgenstein's view can be understood as follows:
    - (a) That the totality of propositions can be surveyed, granted the totality of elementary propositions, by the iteration of the following form of Q&A:
      - Was any of that stuff true?
      - No.
  7. Of course, a proposition doesn't have to be seen as a denial at all; representing it as denial is one way among others to specify by its internal relations to other propositions.
  8. More generally:
    - 5.2: The structures of propositions stand in internal relations to one another.
    - 5.21: We can highlight these internal relations by our means of expression, by representing a proposition as the result of an operation.  $\dots$
    - 5.2341: Negation, logical addition, multiplication, etc., are operations. (Negation reverses the sense of a proposition.)
  9. To summarize: for the  $W$  of the *Tractatus*, the logical forms are forms of judgment, and the forms of judgment constitute internal relations that organize the field of judgments as a whole.