Semantic Theories for Plurals

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Philosophy and Model Theory
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Introduction
First- and higher-order logic contain singular quantifiers, like the existential quantifier (there is something that). But many natural languages have plurals noun phrases and collective predicates.

* John and Bill are young means: John is a young and Bill is young.
* But John and Bill met doesn't mean that John met and Bill met.

How can the semantics of collective sentences be characterized?

* Singularism (most natural language semanticists): Use first- (or higher-) order logic, together with sets or sums

* Pluralism (Boolos, McKay, Oliver & Smiley, Rayo, Yi): Use logics enriched with plural terms, plural quantifiers and collective predicates
* For a semanticist like Link:

Some men carried a piano (together) is true iff
∃s [s is a sum of men ∧ carried-a-piano(s)]
Something is a sum of men and it carried a piano.

* For plural logicians, this is unsatisfactory.
Plural terms, plural quantification and collective predication are primitive, they should be added directly to the logic:

Some men carried a piano (together) is true iff
∃xs [men(xs) ∧ carried-a-piano(xs)]
Some things are men and they carried a piano.
Plural logic
Linnebo 2008, McKay 2006, Rayo 2002, 2006, ...

Singular terms: constants 'a', 'b', ...; variables: 'x', 'y', ...
Plural terms: constants 'as', 'bs', ...; variables: 'xs', 'ys', ...
Operator forming plural terms '&': 'a&b', 'as&bs', ...

Singular existential quantifier '∃x': there is one thing that...
Plural existential quantifier '∃ys': there are some things that...

Predicates: 'P(.)', 'Q(.,..)'
'P(a)', 'P(xs)', 'Q(bs,a)' are well-formed.

Logical relation 'x ∠ ys': x is among (the) ys, x is one of (the) ys
'Q(a&b,c)': a and b together Q c
'∃ys (P(ys) ∧ a ∠ ys)': there are some things ys that together P and a is among the ys
Truth-theory

Variable assignment: some ordered pairs xs that associate
- to each singular variable v an object, eg. z: <v,z>
- to each plural variable ys several objects, eg z, u, t:
  <ys,z>,<ys,u>,<ys,z>

Then satisfaction of a formula $\varphi$ relative to a variable assignment xs, noted 'Sat($\varphi$,xs)', can be implicitly defined by way of axioms.

Model-theory

A domain is given by some things zs.
2 possibilities for interpreting collective predicates:
- use n-ary relations in the metalanguage
- or use ‘superplural’ quantification in the metalanguage
Why is the debate between singularism and pluralism important?

First, ideally, semanticists would like to have the most adequate semantics for plurals.

Second, plural logic has become a very popular tool in the philosophy of mathematics and metaphysics because:
- Plural logic has the expressive and deductive power of monadic second-order logic.
- Unlike second-order logic, plural logic seems ontologically innocent.

But of course, this isn't the case if singularism is right.
In this talk, I will present various singularist positions and discuss arguments against them.

**PLAN**

Introduction

§1. What do we want of a semantics of plurals?

§2. Radical mereological singularism

§3. Liberal mereological singularism

§4. Set singularism

§5. Ontological commitment, or plural innocence?

§6. Absolute generality, or indefinite extensibility?

Conclusion
§1. What do we want of a semantics of plurals?

* A specification of the truth-conditions of sentences containing plurals:
  
  Starting point: paraphrases of various kinds.
  

* A model theory to characterize the notion of logical consequence.

NB: For ease of exposition, I will only specify schematical truth-theories, for fragments of English without quantification. When a full-fledged truth-theory is available, it should be easy to define a corresponding model theory.
§2. Radical mereological singularism

Goodman & Quine (1947) and Goodman (1951), with roots in Leonard & Goodman (1940) and Lesniewski (1919)

Basic idea: the semantic values of plural expressions are (mereological) sums.

*John and Mary surrounded-Castle-Rock* is true iff John+Mary (the sum of John and Mary) surrounded-Castle-Rock.

A plural predication is thus reducible to a singular predication, in which a property is ascribed to one object, a certain sum.
Truth-theory
(schematic, only for a fragment of English without quantification; extendable???)

Subject Verb is true iff Val(Subject) Verb

where 'Verb' is the translation in the metalanguage of Verb

Val(John) = John, Val(Mary) = Mary

Val(John and Mary) = Val(John)+Val(Mary) = John+Mary

Ex: John and Mary surrounded-Castle-Rock is true iff
    John+Mary surrounded-Castle-Rock
What are (mereological) sums?
Many alternative axiomatizations are possible (Simons 1987, Varzi 2009). But the most popular is *classical extensional mereology*.

a overlaps b just in case something is part of a and part of b.
The sum of everything that satisfies Q is the object $s = \sigma[x / Q(x)]$ such that something overlaps s just in case it overlaps something that satisfies Q.

Particular case: $a+b$ is the mereological sum of a and b.

*Axioms:*
- Partial ordering + Strong supplementation
  => The mereology is extensional: objects that have the same parts are identical.
- Axiom schema guaranteeing that the sum of everything that satisfies Q exists whenever something satisfies Q.
First motivation: Multigrade predicates (Leonard & Goodman 1940)

Most predicates in English appear to take different numbers of arguments at the same argument place:

*John and Mary surrounded the castle.*
*John, Mary, and Bill surrounded the castle.*

One simple way to deal with this phenomenon is to use sums. It's always a single object (a certain sum) that surrounds the (Lilliputian) castle.

Second motivation: Nominalism (Goodman & Quine 1947, Goodman 1951)

'We do not believe in abstract entities: classes [ie sets], relations, properties, etc'. 'We renounce them altogether'.
Then they consider some of the nominalist's assets and problems.

Some talk about sets is easily reducible: *The set of cats is included in the set of mammals* is true iff every cat is a mammal.

But what about mathematical discourse generally?

And what about some plural sentences? *There are more dogs than cats.*

Let's look at two problems discussed by friends of plural logic. (For discussion of other problems, cf. Nicolas 2007.)
First problem: John and Bill wrote a book, but their molecules did not

Yet, the mereological sum of John and Bill is identical with the sum of their molecules (Oliver & Smiley 2001).

Intuitive answer: The sums are in fact distinct: the sum of the molecules survives the dispersion of the molecules, while the sum of John and Bill does not since the men do not continue to exist.

We can say that a molecule is part of a man, and that a man is part of two men. But is it the same relation of part? If it is and the relation is extensional, we have a problem.
One way to avoid this problem is to use at least two notions of part.


- Domain of individuals: relation of individual part; atomic individuals like Mary and John; the domain is closed for individual sums.

- Domain of matter: relation of material part; the domain is closed for material sums.

- A function associates to each individual its matter.

We can then coherently deny that the individual sum of the men is identical to the individual sum of their molecules.
Second problem: Counting
Oliver & Smiley (2001), Yi (2005), McKay (2006)

John and Bill are two is true iff John+Bill are or is two.
But how can we get this result, since a sum is one individual?

Answer: Follow Frege (1884) and Geach (1962), like Link (1998).
Counting requires the identification of a sortal or count noun
specifying what is to be counted.

John and Bill are two is understood as John and Bill are two men.
Then, a bit more generally:

*John and Bill are n men* is true iff

i) \( \exists x_1 \ldots \exists x_n \ [ \text{John}+\text{Bill} = x_1+\ldots+x_n \land \forall i \ (\text{man}(x_i)) \land \forall i \neq j \ (x_i \neq x_j) \ ] \)

ii) An analog of condition i) cannot be satisfied for a number \( k \) greater than \( n \)

**Counter-argument:** Suppose the universe contains just a and b (non-overlapping), and their (individual) sum c. a is the (individual) sum of itself, and so is b. So intuitively, these two sentences are true:

*a and b are two individual sums.*

*a, b and c are three individual sums.*

But \( a+b = a+b+c \), so the theory just outlined predicts that *a and b are three individual sums* should be true.
So Link's move of using individual sums works well, except when we proceed to count individual sums themselves.

Possible answer: The metalanguage contains additional logical and mereological notions. Those are not innocent since the semantics associates to any ordinary individuals a *different* object, their individual sum. Therefore, the specialized vocabulary used in the metalanguage *cannot apply to itself*. When doing the semantics of the metalanguage, a different, meta-metalanguage must be used, with a new battery of mereological predicates. And so ad infinitum.

Why should such a hierarchy be more problematic than the hierarchies postulated, among others, by the friends of plural logic (Rayo 2006)?
Nonetheless, life is extremely painful without something (partly) resembling sets, if we want a full-fledged truth-theory (with assignments for quantification) and a corresponding model theory.

§3. Liberal mereological singularism

Sums are still the referents of plural expressions.

But otherwise, set theory is used as it normally is in natural language semantics. The truth-theory now uses sets as denotations for verb phrases:
Truth-theory
(schematic, for a fragment of English without quantification; easily extendable)

Subject Verb is true iff Val(Subject) \in Val(Verb)

Val(John) = John, Val(Mary) = Mary

Val(John and Mary) = Val(John) + Val(Mary) = John + Mary

Ex: John and Mary surrounded-Castle-Rock is true iff
John + Mary \in Val(surrounded-Castle-Rock)
Motivations:
- Multigrade predicates
- Develop semantics as usual
- While sets are abstract entities, sums need not be: the sum of John and Mary is concrete. So while their set could not carry a piano, their sum could.
- Treat plurals and mass nouns (wine, furniture) in similar (though not identical) ways.
What about our earlier problems?
* The men wrote a book, but their molecules did not
We still can't rely on a single extensional mereology.
We can use Link's system, with individual sums for plurals, and
material sums for matter; but other options remain open as well.

* Counting problems: John and Bill are two
Link's strategy is still available, of course, with its consequences
(hierarchy of metalanguages).

Can we proceed differently, since now sets are available, as
denotations of predicates? Can we have a compositional semantics
that works both for John and Bill are two men and John and Bill
surrounded Castle Rock?
§4. Set singularism
Landman (1989), Schwarzschild (1996), among others. The key difference with liberal mereological singularism is this: the referent of a plural expression (what a collective predicate applies to) isn't a sum anymore, but a set.

Truth-theory (schematic, for a fragment of English without quantification; easily extendable)

*Subject Verb* is true iff \( \text{Val}(\text{Subject}) \in \text{Val}(\text{Verb}) \)

\[
\begin{align*}
\text{Val}(\text{John}) &= \{\text{John}\}, \\
\text{Val}(\text{Mary}) &= \{\text{Mary}\} \\
\text{Val}(\text{John and Mary}) &= \text{Val}(\text{John}) \cup \text{Val}(\text{Mary}) = \{\text{John},\text{Mary}\}
\end{align*}
\]

Ex: *John and Mary surrounded-Castle-Rock* is true iff

\[
\{\text{John, Mary}\} \in \text{Val}(\text{surrounded-Castle-Rock})
\]
Motivations:
- Convenience: why use both sets and sums, if we can use only sets?
- The historical connection between plurals and 'classes of things' (ie sets of things). So perhaps what the use of the plural does is to form a set (cf. Russell 1919).

How significant is the difference between set and mereological singularism?
- Pragmatically: according to Landman, in order to characterize the readings of plural sentences, using sets or sums makes no difference.
- Concerning concreteness: Sums of concrete things are concrete, while sets are often said to be abstract.

Well: is there anything in set theory itself that entails that sets of concrete things could not be concrete, have causal powers?

- Ontologically: It is sometimes said that a sum of men is identical to its parts (the men), while a set of men is distinct from its members. However, composition as identity is problematic if taken literally; cf Sider (2007, submitted) for very weird consequences of the view.
§5. Ontological commitment, or plural innocence?
Yi (2005), McKay (2006)

For the singularist:
*John and Mary surrounded Castle Rock* is true iff the sum / set of John and Mary ...
The right-hand side implies that there exists a sum / set.

But the original, English sentence would not. More generally, plural sentences (and hence, plural logic) would be ontologically innocent. (NB: This alleged innocence is an important reason why plural logic seems so attractive in metaphysics.)
For the friend of plural logic:

Truth-theory (transparent)
(Schematic, for a fragment of English without quantification; easily extendable)

*Subject Verb* is true iff *Subject Verb*

Ex: *John and Mary surrounded Castle Rock* is true iff
    
    John and Mary surrounded-Castle-Rock

where ‘John and Mary’ is a plural term and there are axioms relating the behavior of 'and' and 'among' in the metalanguage (John is among John and Mary, Mary is among John and Mary, and nothing else is)
First answer: The purpose of such bi-conditionals is to characterize inferential relations between various sentences of English. The metalanguage used brings its own commitments (to sums / sets), independent of those of sentences of English.

Response: Ideally, a semantic theory for natural language should characterize inferential relations in a transparent fashion, and in a way which makes clear what the ontological commitments of English sentences are.

Well, is that correct?

But also:
Second answer: The ontological commitments of a sentence and the inferences that can be drawn from it are difficult to assess. One should not rely only on hunches and intuitions.

Sums and sets are theoretical notions, and so it's indeed not clear whether an inference to their existence holds.

But there are similar inferences that seem quite natural: 
\textit{The men surrounded the castle.}
\Rightarrow \textit{A group of men surrounded the castle.}
And perhaps a group of men is a sum or set of men, this very fact failing to be transparent to ordinary speakers.
Parallel case: If Davidson is right, the semantics of many sentences involves existential quantification over events: 

*John walked* is true iff $\exists e \ (\text{walking}(e) \ & \ \text{agent}(e, \text{John}))$

But the existence of this event isn't clear on the basis of intuitions about the meaning of *John walked*.
If Davidson's semantics is attractive, its because of its theoretical merits, not solely or primarily because of intuitions.

Also, on a simpler analysis, *John walked* is true iff $\text{walked}(\text{John})$. Because of this, for Frege, the sentence is committed both to the object John and to the function walked(). And why not indeed?
§6. Absolute generality versus indefinite extensibility
Cf. Linnebo (2008, §4.3)

In mathematics and its philosophy, we may want to talk about absolutely all sets:

*Any set satisfies the axioms of set theory.*

*There are some sets such that a set is one of them just in case it is not a member of itself.*
(i.e. *There are some sets which are all and only the non-self-membered sets*)

What semantics can we propose for the 'Russelian' sentence?
First, using sets, we may propose:

\[ \exists y \, \forall x \, (x \in y \iff x \notin x) \]

But if the quantifiers \( \exists y \) and \( \forall x \) range over the same sets and if \( y \) exists, we get a contradiction:

\[ y \in y \iff y \notin y \]

So if the sentence does range over all sets, and it is translated as proposed, then the sentence must be false (\( y \) cannot exist), while it seems to be genuinely true.

One response is in terms of indefinite extensibility. (Cf. Dummett 1981, Glanzberg 2004, 2006)

The translation proposed is adequate, but the range of the quantifier \( \exists y \) is bigger than that of \( \forall x \).
So Russell's paradox shows that, contrary to first appearances, we cannot quantify over absolutely all sets. The concept of set is indefinitely extensible: whenever we have formed a conception of a certain range of sets, we can define a set that isn't in that range.

Criticism: the view is hard to state and perhaps self-refuting (Williamson 2003). But of course, this is hotly disputed, cf. notably Glanzberg.

So a second response translates the sentence in terms of sums:
\[ \exists y \; \forall x \; (\text{set}(x) \rightarrow (x \leq y \leftrightarrow x \notin x)) \]
There is a sum \( y \) which is the sum of all the sets that don't belong to themselves.

If we make no special assumptions about sets and sums, then there is no contradiction.
Conclusion

* Radical mereological singularism:

- Because it renounces sets altogether and uses only sums, the development of a full-fledged truth-theory and a corresponding model theory seems impossible.

- One should not use a single, extensional mereology. Inspired by Link, one may use two mereologies.

- Counting problems (and similar ones) lead this form of singularism to postulate an infinite hierarchy of metalanguages. In itself, this doesn't seem so bad, as most theories are lead, because of paradoxes, to similar hierarchies. But once the hierarchy is in place, one wonders why not just use sets, or perhaps, sums and sets?
Liberal mereological singularism: sums are the referents of plural expressions, but sets are employed as the denotations of predicates.

Set singularism: sets are the referents of plural expressions

How different are they from one another?

Two other problems for singularist approaches:
- Their ontological commitment to sums or sets: the argument isn’t convincing.
- The threat of paradox if quantification over absolutely everything is possible: all depends on whether quantification over absolutely everything is possible, or whether some concepts (like set) are indefinitely extensible.
* So what about plural logic?

Plural terms, plural quantification and collective predication are taken to be primitive.

Plural logic, if genuine, is attractive because of:
- its power (similar to monadic second order logic)
- its transparency and its ontological innocence.
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