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Etchemendy’s objections to Tarski’s account of the notion of logical consequence

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Ce qui suit est un simple canevas ayant servi de support à l’exposé oral
• One says commonly that a sentence $X$ is a logical consequence (LC) of a set $K$ of sentences, iff, *in virtue of logic alone*, it is impossible for the sentences of $K$ to be all true without $X$ being true as well.

• We may intuitively understand what is meant by such a definition, but we still should explicit what means “*in virtue of logic alone*” and how to understand “*it is impossible…*”.

• I will try to explain how Tarski understood those expressions
Different expressions

• If $X$ is a logical consequence of $K$, we say *also* that $X$ follows from $K$, or $K$ implies or entails $X$ (implication is the converse relation of that of consequence), or also that one may correctly infer the *truth* of $X$ from the truth of the sentences of $K$.

• Network of concepts (truth, implication, entailment, inference, consequence, proof, etc.) which indicates the multiple aspects of the notion of logical consequence
• N.B.

- Being derivable or being a theorem, and being a logical consequence are properties of propositions.

- Validity is a property of arguments. One argument with premises $K$ and conclusion $X$ is valid iff $X$ is a logical consequence of $K$.

• In the frame of classical logic, which is T’s frame, we distinguish two notions of LC. One is involved in the process of derivation (syntactical notion); the other points to the alternative concept of deducibility (semantic notion as used informally before T and defined by T). Cf. Sundholm 2002.
Everyone knows Tarski’s definition of LC, which was central to the new perspective opened up, the semantic and model-theoretic perspective. Yet I shall recall it.

If one wants a clear summary of T’s 1936 paper, one may read G. Sher 1996, p. 655-656.

My aim here is historical; it consists in rereading T’s 1936 paper in order to get an idea of the relevance of the very many readings of it, some of them being critical and negative, some others being a defence or a complement to T’s views and formulations. In particular, Etchemendy’s 1990 book launched a controversy which is not yet over. This is why I will consider especially Etchemendy’s objections.
Formal investigation and common usage

• “The concept of logical consequence is one of those whose introduction into a field of strict formal investigation [emphasis mine] was not a matter of arbitrary decision on the part of this or that investigator; in defining this concept efforts were made to adhere to the common usage of the language of everyday life. …. With respect to the clarity of its content the common concept of consequence is in no way superior to other concepts of everyday language. Its extension is not sharply bounded and its usage fluctuates. Any attempt to bring into harmony all possible vague, sometimes contradictory, tendencies which are connected with the use of this concept, is certainly doomed to failure. We must reconcile ourselves from the start to the fact that every precise definition of this concept will show arbitrary features to a greater or less degree.” (T 1936, p. 409)
The “common concept” and its formal representation by the concept of proof

• “Even until recent years many logicians believed that they had succeeded, by means of a relatively meagre stock of concepts, in grasping almost exactly the content of the common concept [emphasis mine] of consequence, or rather in defining a new concept which coincided with the common one…Thanks to the progress of mathematical logic we have learnt, during the course of recent decades, how to present mathematical disciplines in the shape of formalized deductive theories. In these theories, as is well known, the proof of every theorem reduces to single or repeated applications of some simple rules of inference – such as the rules of substitution and detachment….Logicians thought that these few rules of inference exhausted the content of the concept of consequence….” (T 1936, pp. 409-410).
Scientific semantics

“I should like to sketch here a general method which, it seems to me, enables us to construct an adequate definition of the concept of consequence for a comprehensive class of formalized languages” … The ideas involved in this treatment will certainly seem to be something well known…Nevertheless it seems to me that only the methods which have been developed in recent years for the establishment of scientific semantics, and the concepts defined with their aid, allow us to present these ideas in an exact form”. (T 1936, p. 414).
The “proper concept of consequence”/alternative formal definition

• “It is perhaps not superfluous to point out in advance that in comparison with the new [semantic] – the old [syntactic] concept in no way loses its importance. This concept will probably always have a decisive significance for the practical construction of deductive theories…It seems, however, that in considerations of general theoretical nature the proper [semantic] concept of consequence must be placed in the foreground”. (T 1936, p. 413).

• T’s prediction might seem fulfilled since some logical systems do not allow for complete axiomatization, e.g. full second-order logic, with quantification over all subsets of the universe.
Intuitive standpoint and formal relation

• “Consider any class K of sentences and a sentence X which follows from this class. From an intuitive standpoint, (1) it can never happen that both the class K consists of only true sentences and the sentence X is false. (2) Moreover, since we are concerned here with the concept of logical, i.e., formal consequence, and thus with a relation which is to be uniquely determined by the form of the sentences between which it holds, this relation cannot be influenced in any way by empirical knowledge, and in particular by knowledge of the objects to which the sentence X or the sentences of class K refer. The consequence relation cannot be affected by replacing designations of the objects referred to in these sentences by the designations of any other objects.”
(1) And (2) jointly expressed in the statement (F)

- (F) “If, in the sentences of the class K and in the sentence X, the constants – apart from purely logical constants – are replaced by any other constants (like signs being everywhere replaced by like signs), and if we denote the class of sentences thus obtained from K by K’, and the sentence obtained from X by X’, then the sentence X’ must [emphasis mine] be true provided only that all sentences of the class K’ are true” (T 1936, p. 414, [Tarski’s italic]).

- This formulation is the same as Bolzano’s formulation of logical deducibility (See Sebestik Stanf. Encyc. of Phil.)
Remarks on (F)

• **I.** In (F) occurs the term ‘true’, not the term ‘proof’; no reference to the application of some rules of inference.

• **II.** (F) is meant to jointly express
  - 1. **truth preservation** from premises to conclusion
  - 2. **formality** in accordance with the intuitive concept and with the ordinary usage. ‘Formal’ is not restricted to syntactic concepts (1 and 2 are the two “adequacy conditions”). Formality is bound with the problem of defining logical constants. I will not touch this problem.

• **III.** (F) resorts to a substitutional account of logical truth (cf. Bolzano’s conception)

• **IV.** (F) is a necessary condition

• **V.** According to T, (F) could be regarded as sufficient only if the designations of all possible objects occurred in the language, i.e. in the Carnapian perspective of a “general syntax” dealing with a language which possesses a sufficient stock of extra-logical constants
N.B.

- Some logicians, e.g. Garcia Carpintero (1993, p.126-127), G. Sher (1996), mention that T did not give a proof showing that (F) is a necessary condition. Garcia Carpintero thinks that only in a vague sense one can prove that (F) captures the « formality » and « the modal not » of the intuitive notion of LC. Others, especially, Gómez-Torrente (200, p.531-533), suggest that T probably had a proof for his notion of LC, which is to be understood as less strict than our usual model-theoretic notion of LC.
• But T judged the universalistic perspective as “fictitious” (p. 416) and took a stand that he wanted “independent of the richness in concepts of the language being investigated” (p. 417). Hence condition (F) is in general not sufficient.

• For a definition independent of the universalistic perspective, T considers the concept of satisfaction, which “must always be relativized to some particular language” (p. 416) and the concept of model defined in terms of satisfaction and “in just the sense one usually speaks of the models of an axiom system of a deductive theory” (p. 417). He switches also from a substitutional account (statement (F)) to a semantic account.
Definition of logical consequence through the notion of model

• The sentence $X$ follows logically from the sentences of the class $K$ iff every model of the class $K$ is also a model of the sentence $X$. (T 1936, p. 417, [Tarski’s italic]).

Tarski stresses that this definition (let us name it (D)) is equivalent to that of Carnap. He states that

1. A class $K$ is contradictory if it has no model or iff every sentence follows from it

2. A class $K$ can be called *analytical* if every sequence of objects is a model of it or iff it follows from every class of sentences (in particular from the null class).
N.B.

- There are discussions about the link between (F) and (D). T wrote that, on the basis of this definition (D), “it can be shown that the condition (F) is necessary” (p. 417).
- G. Sher (1996) thinks that T never gave the proof.
- Gómez-Torrente (1996, p. 130, 2000, p. 532-535) claims that one can actually prove (F) on the basis of the definition (M) of model given right before (D), and he reconstructs a sketch of the proof that T presumably had in mind.
- (M): “an arbitrary sequence of objects which satisfy every sentential function of the class L’ will be called a *model* of the [corresponding] class L ...(in just the sense one usually speaks of models of an axiom system of a deductive theory)”.
First observations

- T’s conception of validity and logical consequence in his 1936 paper does not involve any agent who thinks or establishes that a sentence X is or is not a logical consequence of a set K.
- No reference to the problem of how to determine validity or with how people have established validity in the past.
- Hence, no direct bearing on the heuristics, epistemics, or pragmatics of consequence, or with the context of discovery of the validity of a given argument (See Tarski’s text – diapo 11- and e.g. Sagüillo 1997, p. 220, Jané 2006). Compare e.g. with Padoa’s position, “Logical introduction to any deductive theory”, in Van Heijenoort 1967, p.120-121.
I don’t follow Etchemendy, who argues that T’s method misses the target: connecting the intuitive and the formal side of LC. But I understand that Tarski’s method is not the only possible choice. Etchemendy (1990, p. 94) demands that “arguments declared valid carry with them an independent guarantee of truth preservation, whether modal or epistemic or semantic”, and he proposes in fact neither a modal nor an epistemic guarantee, but an alternative approach to semantics (“representational semantics” : theory of “x is true in W”, where W is a variable whereas interpretational semantics is a theory of “x is true in L” for some range of languages L).

Other alternative approaches to the relation of logical consequence which deal formally with questions about the justifications of logical consequence and about how we come to know logical truths:

- Prawitz’s conception of logical consequence as a relation between rules of deduction (Synthese 62, 1985, 153-171) or
- Martin Löf’s proof-theoretic semantics
• Prawitz: T’s analysis of truth “makes no distinction between logical sentences (containing only logical constants) and factual sentences (containing also descriptive constants). The effect is that a logical sentence is understood as logically true just in case it is true in the same sense as factual sentences are true. In other words, no analysis is made of the necessity involved in logical truth - not to mention that no answers are attempted to questions like what is the ground for a universal truth, or how can we come to know, even with certainty, that a logical truth is true in all domains”.

Comments

1. T’s frame was the study of the structure of “deductive theories”, i.e. of interpreted formal systems, not the study of language universally.

2. T’s formal concept of logical consequence was not elaborated for replacing but for completing the formal concept of derivability; it was introduced as a new formal concept, which brings about new ways of reasoning.

3. The link between proof and model was made in the mathematical and logical practice since at least the XIXth century, notably and explicitly in Padoa’s method for proving that a proposition $X$ is not derivable from a set $S$ of axioms (See “Logical introduction to any deductive theory”, in Van Heijenoort 1967)
4. T calls derivability (and not deducibility, as is written in most papers about T 1936) “the concept of consequence formalized” in such a way that it refers to the formalized concept of proof (p. 410 and p. 413 footn. 2). ‘Derivability’ is used in accordance with Carnap’s usage.

“In his extremely interesting book [Logische Syntax der Sprache, 1934], Carnap R., the term (logical) derivation or derivability is applied to the old concept of consequence as commonly used in the construction of deductive theories, in order to distinguish it from the concept of consequence as the proper concept… He also emphasizes – to my mind correctly—the importance of the proper concept of consequence and the concepts derived from it, for general theoretical discussions”.

• 5. T’s arguments for claiming that the content of what was for him the intuitive concept of logical consequence was not exhausted by the concept of provability:
  - 1. Example of a $\omega$-incomplete theory
  - 2. Gödel’s incompleteness theorem

• But
  – See e.g. Sagüillo 1997 for an analysis of the reasons why T was certain of the validity of all $\omega$-arguments, while it can be shown that some $\omega$-arguments are not valid.
  – See e.g. Prawitz’s view about the impact of Gödel’s incompleteness theorem. Naturally, T’s way was not that of natural deduction (Prawitz 1985).

• 6. Tarski uses the term ‘sentence’, rather than ‘proposition’, referring to expressions which are distinguished by their purely structural character, wffs in our language. Strictly speaking that are propositions expressed by sentences, which are true or false (as noticed by Sundholm 2002, sentences are the metamathematical counterparts to propositions)

• 7. As noticed by some readers, T is using three notions of consequence:
• (a) the “common concept” [der übliche Begriff], informally used
• (b) the concept used in ”the formalized deductive theories”: the
  (syntactic) concept of derivability, that, according to him, does not
  exhaust the content of the common/ordinary concept
• (c) the “proper concept” that he was formally defining in the frame of his
  “scientific semantics”

➢ The common/ordinary concept belongs not to the everyday language
  but to the portions of natural languages that logicians and
  mathematicians used in their practices (Gómez-Torrente 1996, Jané
  2006 among others – I am surprised that there still is a need to stress
  this aspect, which is obvious in T’s stuff).
➢ T’s investigation does not pertain to studies of natural languages as
  such nor to semantics as philosophy of language.
  That does not mean that T had no interest in philosophical questions.
  That means only that he considered philosophical questions difficult
  and beyond any area where we could hope to give them exact
  answers. In this sense, his analysis is actually “reductive”, as
  Etchemendy reproaches to it.
Main Criticisms

• 1. Is T’s common/ordinary/intuitive concept of logical consequence pre-theoretic?
• 2. The necessity reading of logical consequence
• 3. The problem of distinguishing logical constants from extra-logical constants.
• 4. The problem of variation of the domain
• 5. The lack of a theory of meaning

• It already have said enough on point 1. Answer : No
• I shall not discuss the points 3 and 4 (cf. Mancosu 2010)
• I shall discuss points 2 and 5 only from Etchmendy’s perspective
Modal reading of T’s paper/Analysis of modality

First of all, Etchemendy criticizes in general the “doctrine of logic as form”, *Linguistics and Philosophy* 6, 1983,319-334 : “So far I have seen little reason to think that form has much to do with logic at all”. Etchemendy considers forms as persistent patterns ruling our linguistic uses. But this empiricist credo is counteracted by a metaphysical grounding of these uses.

- As what concerns T’s logical consequence, Etchemendy (among others) claims
  - 1. that this concept involves an implicit modal aspect, which is not treated in T’s account, and
  - 2. that this modal aspect is essential and that an account of LC must capture this essential feature. In other words the adequacy condition (1) in (F) must be understood as wording not truth preservation but **necessary** truth preservation
If $X$ is a logical consequence of $K$, then not only it is the case that not all of the elements of $K$ are true and $X$ is false, but also this is necessarily the case. For E “the most important feature of logical consequence, as we ordinarily understand it, is a modal relation that holds between implying sentences and sentence implied.” (E 1990, p. 81)
“Tarski’s fallacy”

3. Moreover, Etchemendy claims that T made a fallacy in proving at most
   • Necessarily (if $P$ then (if $Q$ then $R$)),
   • while he should have proved
     • If $P$ then necessarily (if $Q$ then $R$),
where
   - $P$ is: ‘$X$ is a consequence of $K$ according to T’s definition’
   - $Q$: ‘All the sentences in $K$ are true’
   - $R$: ‘$X$ is true’.

Also according to E, T made an illicit shift in the position of the modality
• According to E, Tarski has defined material consequence not LC
• Gila Sher claims also that T actually asserted that logical consequence is necessary but she thinks that the claim that Tarski committed “Tarski’s fallacy” is not substantiated (1996): T did not indicate in any way what the proof of condition (F) was.
• Gómez-Torrente (“On a Fallacy attributed to Tarski”, History and Philosophy of Logic 19, 1998, 227-234, especially pp. 228-230) has another reading.

According to him, Tarski did give a hint for a proof of (F) and he used modal expressions (‘can never’, ‘must’, etc.) to stress formal generality, not to express a modal relation between premises and conclusion. If an argument of a certain form is valid, then all arguments of that form are also valid.

“Tarski is not talking about any sort of necessary truth, or about truth knowable a priori, or about analytic truth. He is talking about truth full stop […] the plain, simple, unqualified, unvarnished truth about sets”.

Ray 1996 (pp. 642-646) rightly argues that condition (F) is lacking “any modal force”.

Sagüillo 1997 (p. 237) rejects the modal interpretation and is inclined to say that those expressions (‘must’, ‘necessarily’) are “mere vestiges of the ordinary way of referring to the pre-formal notion of logical consequence.” This might be the case, but if it really were, it would not dispense us with the need for explaining why Tarski did keep these “vestiges”.

For Gómez-Torrente 1998, the “modal” expressions are indicating a universal quantifier over a certain domain

García-Carpintero 2003 (p. 167) gives a way out: we are today familiar with semantic precise quantificational explanations, in terms of possible worlds, of intuitive modal concepts. So formal generality” and modality are not opposite.
• Anyway, in T’s general perspective about the relations between semantics and philosophy, it is hard to interpret those modal wordings as really indicative of an effective philosophical commitment.

• Indeed, an indication is given by Carnap’s Autobiography: Carnap suggests that Tarski was similar to Quine in his avoidance of modal notions.

• Tarski did not intend to deal with modality (in his paper on LC).

• To substitute, as E does, a semantics grounded on not totally clear metaphysical views to a semantics which was elaborated to leave apart metaphysics and to give precise tools to mathematical reasoning does not seem to me a good idea.
In one word one thing is to see an implicit modality in T’s definition, or, more generally, in the usual definition of logical consequence. A totally different job is to give account – a formal one – of the necessity element presumably involved in the relation of logical consequence. It is clear that T did not intend to give account of this aspect in terms of modality.
Conceptual mistake: confusing the symptoms of LC with their cause

- “For example, the classical semantics for propositional logic may not provide a fully grounded explanation of the principle of excluded middle, but it does explain why, given this basic assumption [emphasis mine], a complex sentence like

\[ \neg (P \land (\neg P \lor (Q \land R))) \lor Q \] is necessarily true.” E 2008

- E argues that the truth preservation results from the logical consequence, not the other way around. “model theory, properly understood, does not yield an analysis of the logical properties, but presupposes them”.

- (1) It is true that all instances of a valid argument (e.g. MP.) preserve truth because it is a logically valid argument form.

- (2) It is false that a valid argument is logically valid because its instances preserve truth.

- According to E. T’s definition is based on the false assumption (2).

- I don’t agree. In my mind, T was not questioning the validity of any rule of inference; he actually assumed this validity.
What T did was not to prove the validity of modus ponens or of the law of excluded middle, but to state that a valid argument preserves truth modulo the assumed truth of some presupposed logical laws (MP, LEM). Verifying truth preservation from premises \( K \) to consequence \( X \) does not prove the validity of, let us say, MP, but does prove the validity of some particular argument instantiating MP. T’s definition was coined for giving a formal means to apply the rule, just as formal derivation is a means to obtain theorems from axioms or assumptions on the basis of some assumed rules of inference.

Etchemendy’s reproach comes from a foundational perspective, which was not, at least not firmly nor constantly T’s own perspective. Moreover one can argue (van Benthem 2002) that any foundational perspective involves some circularity.
Theory of meaning?

• The core of some recurring points of Etchemendy’s criticisms (and before him of many other logicians - Dummett, Martin-Löf, Prawitz-) comes down ultimately to pointing out the lack of a semantic theory of meaning in T’s model-theoretic view.

• And there is indeed no philosophical elaboration of the relations between meaning and language in T’s work, although he was referring to the content of the term ‘meaning’ in very many occasions, not always in a uniform way (see my contribution to the book Logicism, Intuitionism, Formalism, edited by Sten Lindström & alii), Springer, Synthese Library, 2009.
Some quotations from Etchemendy’s book

- “the “crucial guarantee”… “emerges from semantic characteristics of the language”.
- “The semantics shows precisely how the logic of the language arises from the meanings of its constituent expressions, modulo any basic logical assumptions incorporated into the models themselves. For example, the classical semantics for propositional logic may not provide a fully grounded explanation of the principle of excluded middle, but it does explain why, given this basic assumption, a complex sentence like

\[ \neg (P \land (\neg P \lor (Q \land R))) \lor Q \]

is necessarily true.”
“The crucial feature of modus ponens is that we can recognize that all of its instances preserve truth without knowing the specific truth values of the sundry instances. My own view is that we recognize this by virtue of the meaning [emphasis mine] of the expression ‘if…then’ and our knowledge of how the remaining constituents can contribute to the truth values of the premises and conclusion.. “ E 2008 (Reflections on consequence, in Patterson 2008, OUP)
Precisions on Tarski’s use of ‘meaning’

- One the one hand:
  - Logic, wrote Tarski, is “a discipline which analyzes the meaning of the concepts shared by all the sciences, and states the general laws ruling those concepts.” Tarski 1960, p. XII (here T meant not only the deductive sciences, but also the experimental sciences). The scope of logic is even wider, since Tarski aimed to create «a unified conceptual apparatus which would supply a common basis for the whole of human knowledge». See S. Feferman 2004

- On the other hand:
  - ‘meaning’ is not a semantic term in T’s “scientific semantics”; there is no metamathematical counterpart of the intuitive notion T is referring. There is no theory of meaning in T’s semantics. Cf discussion with Kokoszynska, who proposed that “the concept of truth – in one of its interpretation – should be relativized to the concept of meaning. Would not be simpler to relativize it to the concept of language, which is clearer and logically less complicated than the concept of meaning?” (Collected Works, IV, p. 701).
More on Etchemendy’s position

- Finally, “Logical truth and logical consequence are just two persistent patterns emerging from the meaning of sentences of the considered language.”

Indeed, Etchemendy’s representational semantics “should be seen as a method of approaching the empirical study of language” (1990, p. 25). This project has clearly a totally different perspective from Tarski’s own perspective.
Assessment of E’s criticisms

• When E points out modality, knowledge, information, indexicality, temporality, meaning as fundamental aspects of our pre-theoretic idea of logic, although he disregards T’s own aims, he is drawing attention to aspects that are worth considering.

• When E argues that studying second-order logic, modal operators (Carnap, Kanger, Kripke), epistemic notions (Hintikka), indexicals (Kaplan) is doing logic, he is naturally right.

• When he says that T’s account leads to a “limiting” view of logic, it would have been fair to say also that T was not unaware of the bounds of his study (see quotation of diapo 7). So far as I know, T never wrote that there was no other interesting and logically relevant aspects than those he was taking into account.
• Most of E’s criticisms rest on a misconstruction of T’s account of LC

• One should only recognize that E really pinpoints an important fact which impacts not only T’s view, but, more generally, the view of classical logic (propositional calculus + first-order languages + assumption of certain laws, especially the law of excluded middle). Just E was not the first philosopher/logician to pinpoint the fact that classical logic is not all logic.
Keeping model-theoretic tools, changing their conceptual analysis

• E’s claim is indeed the following: “I will sketch what I consider the proper understanding of model-theoretic semantics and its relation to the pre-theoretic notions of logical consequence and logical truth.”

• E recognizes that the shift from interpretational semantics to representational semantics does not ruin the efficiency of the technical tools of model-theory. “the critique is not aimed at model-theoretic techniques, properly understood”

• the model-theoretic techniques are still working well, but we have to change our semantic or philosophic or conceptual analysis of those techniques. This change is meant to open up new areas of study that might seem precluded by T’s analysis.
• But the model theory of E’s semantics is couched on a background founded on general metaphysics. And the constraints on metaphysical models are more involved than those on logical models (cf. Sher 1996, p. 659-661)
Conclusions

• If E’s harsh criticisms (fallacy, conceptual mistake, etc.) were founded, they would have a blasting impact not only on T’s account of LC, but also on our whole understanding of parts of contemporary logic. But they are not.

• If we are interested in a completely alternative philosophical theory of meaning, then we can refer to Dummett’s work or to Martin-Löf’s elaboration on meaning.

• If we want to have examples of how meaning can be taken into account in a operational logical analysis, then we may refer to works developed on a similar line as that of natural deduction or that of intuitionistic type theory.
Bibliography

- Benis Sinaceur H
- Bonnay D
  2006 Logicality and Invariance, The Bulletin of Symbolic Logic 14 (1), 29-68
• Etchemendy J
• 1990 *The concept of logical consequence*, Harvard University Press.
• Feferman S
• Gárcia-Carpíntero Sánchez-Miguel S
• Gómez-Torrente M
• 1998 On a fallacy attributed to Tarski, *History and Philosophy of logic* 19, n° 4, 227-234.
• Jané I
• Lindström S, Palgren E, Segerberg K, and Stolenberg-Hansen V (eds)
• Mancosu P
• 2010 Fixed domain vs variables domain conceptions of logical consequence, *Philosophy Compass*.
• Martin-Löf P.
• McGee V

• Padoa A
  1900 Logical introduction to any deductive theory, in Van Heijenoort 1967.

• Patterson D
• Prawitz D
• Ray G
• Sagüillo J.M.
• Sher G
• Tarski A
• 1936 On the concept of logical consequence, ibidem, 409-420.
• 1944 The Establishment of scientific semantics, ibidem, 401-408.
• 1960 Introduction to Logic and to the Methodology of Deductive Sciences, Oxford University Press, New York (first ed. 1941).

• Van Heijenoort J
FIN